Results on MNIST and Contagio. Figure 2(b) shows the results of BagFlip on MNIST and Contagio. When fixing R, the certified accuracy for the backdoor attack is much smaller than the certified accuracy for the trigger-less attack (Figure 1 and Figure 3 in Appendix E.2) because backdoor attacks are strictly stronger than trigger-less attacks. BagFlip cannot provide effective certificates for backdoor attacks on the more complex datasets CIFAR10 and EMBER, i.e., the certified radius is almost zero. **BagFlip can provide certificates against backdoor attacks on MNIST and Contagio, while BagFlip's certificates are not effective for CIFAR10 and EMBER.**

7.3 Computation Cost Analysis

We discuss the computation cost of BagFlip on the MNIST dataset and compare to other baselines.

Training. The cost of BagFlip during training is similar to all the baselines because BagFlip only adds noise in the training data. BagFlip and other baselines take about 16 hours on a single GPU to train $N = 1000$ classifiers on the MNIST dataset.

Inference. At inference time, BagFlip first evaluates the predictions of N classifiers, and counts how many classifiers have the majority label (N_1) and how many have the runner-up label (N_2). Then, BagFlip uses a prepared lookup table to query the radius certified by N_1 and N_2. The inference time for each example contains the evaluation of N classifiers and an O(1) table lookup. Hence, there is no difference between BagFlip and other baselines.

Preparation. BagFlip needs to prepare a table of size O(N^2) to perform efficient lookup at inference time. The time complexity of preparing each table entry is presented in Sections 5 and 6. On the MNIST dataset, BagFlip with the relaxation proposed in Section 6 ($\delta = 10^{-4}$) needs 2 hours to prepare the lookup table on a single core. However, the precise BagFlip proposed in Section 5 needs 85 hours to prepare the lookup table. Bagging also uses a lookup table that can be built in 16 seconds on MNIST (Bagging only needs to do a binary search for each entry). FeatFlip needs approximately 8000 TB of memory to compute its table. Thus, FeatFlip is infeasible to run on the full MNIST dataset. FeatFlip is only evaluated on a subset of the MNIST-17 dataset containing only 100 training examples. RAB does not need to compute the lookup table because it has a closed-form solution for computing the certified radius.

BagFlip has similar training and inference time compared to other baselines. The relaxation technique in Section 6 is useful to reduce the preparation time from 85 hours to 2 hours. Even with the relaxation, BagFlip needs more preparation time than Bagging and RAB. We argue that the preparation of BagFlip is feasible because it only takes 12.5% of the time required by training.

8 Conclusion, Limitations, and Future Work

We presented BagFlip, a certified probabilistic approach that can effectively defend against both trigger-less and backdoor attacks. We foresee many future improvements to BagFlip. First, BagFlip treats both the data and the underlying machine learning models as closed boxes. Assuming a specific data distribution and training algorithm can further improve the computed certified radius. Second, BagFlip uniformly flips the features and the label, while it is desirable to adjust the noise levels for the label and important features for better normal accuracy according to the distribution of the data. Third, while probabilistic approaches need to retrain thousands of models after a fixed number of predictions, the deterministic approaches can reuse models for every prediction. Thus, it is interesting to develop a deterministic model-agnostic approach that can defend against both trigger-less and backdoor attacks.

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Checklist

1. For all authors...
    (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
    (b) Did you describe the limitations of your work? [Yes] See Section 8.
    (c) Did you discuss any potential negative societal impacts of your work? [N/A]
    (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
    (a) Did you state the full set of assumptions of all theoretical results? [Yes]
    (b) Did you include complete proofs of all theoretical results? [Yes] We provide them in supplementary materials.
3. If you ran experiments...
    (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes]
    (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
    (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No] As our results are aggregated from thousands of models with the same statistical confidence level, we do not report the error bars.
    (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes]
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
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5. If you used crowdsourcing or conducted research with human subjects...
    (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
    (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
    (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

A Probability Mass Functions of Distributions for FL$_s$, F$_s$, and L

Distribution μ for F$_s$. The distribution $\mu(D,\mathbf{x})$ describes the outcomes of $\dot{D}$ and $\dot{\mathbf{x}}$. Each outcome $\dot{D}$ and $\dot{\mathbf{x}}$ can be represented as a combination of 1) selected indices $w_1,\dots,w_k$, 2) smoothed training examples $\mathbf{x}'_{w_1},\dots,\mathbf{x}'_{w_k}$, and 3) smoothed test input $\mathbf{x}'$. The probability mass function (PMF) of $\mu(D,\mathbf{x})$ is,

$$p_{\mu(D,\mathbf{x})}(\dot{D},\dot{\mathbf{x}}) = \frac{\rho^{(k+1)d}}{n^k}\left(\frac{\gamma}{\rho}\right)^{\sum_{i=1}^{k}\|\mathbf{x}'_{w_i}-\mathbf{x}_{w_i}\|_0+\|\mathbf{x}'-\mathbf{x}\|_0}, \tag{10}$$

where d is the dimension of the input feature, $\gamma = \frac{1-\rho}{K}$, $K+1$ is the number of categories, and $\|\mathbf{x}'-\mathbf{x}\|_0$ is the l_0-norm, which counts the number of different features between $\mathbf{x}$ and $\mathbf{x}'$. Intuitively, Eq 10 is the multiply of the two PMFs of the two combined approaches because the bagging and the noise addition processes are independent.

Distribution μ' for FL$_s$. For FL$_s$, we modify μ to μ' such that also flips the label of the selected instances. Concretely, $\dot{D}$ becomes $\{(\mathbf{x}'_{w_1},y'_{w_1}),\dots,(\mathbf{x}'_{w_k},y'_{w_k})\}$, where y' is a possibly flipped label with γ probability. And the PMF of $\mu'(D,\mathbf{x})$ is,

$$p_{\mu'(D,\mathbf{x})}(\dot{D},\dot{\mathbf{x}}) = \frac{\rho^{(k+1)d+k}}{n^k}\left(\frac{\gamma}{\rho}\right)^{\sum_{i=1}^{k}\|\mathbf{x}'_{w_i}-\mathbf{x}_{w_i}\|_0+\mathbb{1}_{y_{w_i}\neq y'_{w_i}}+\|\mathbf{x}'-\mathbf{x}\|_0}, \tag{11}$$

where $\mathbb{1}_{y_{w_i}\neq y'_{w_i}}$ denotes whether the label y'_{w_i} is flipped.

Distribution μ'' for L. For L, we modify μ to μ'' such that it only flips the label of the selected instances. Also, it is not necessary to generate the smoothed test input because L cannot flip input features. Concretely, $\dot{D}$ becomes $\{(\mathbf{x}_{w_1},y'_{w_1}),\dots,(\mathbf{x}_{w_k},y'_{w_k})\}$, where y' is a possibly flipped label with γ probability. And the PMF of $\mu''(D)$ is,

$$p_{\mu''(D)}(\dot{D}) = \frac{\rho^k}{n^k}\left(\frac{\gamma}{\rho}\right)^{\sum_{i=1}^{k}\mathbb{1}_{y_{w_i}\neq y'_{w_i}}} \tag{12}$$

B Neyman–Pearson Lemma for the Multi-class Case

In multi-class case, instead of Eq 4, we need to certify

$$\forall \widetilde{D}\in S_r^{\pi}(D),\ \widetilde{\mathbf{x}}\in\pi(\mathbf{x},y)_1.\ \Pr_{o\sim\widetilde{\mu}}(A(o)=y^*) > \Pr_{o\sim\widetilde{\mu}}(A(o)=y'), \tag{13}$$

where y' is the runner-up prediction and p' is the probability of predicting y'. Formally,

$$y' = \underset{y\neq y^*}{\text{argmax}}\Pr_{o\sim\mu}(A(o)=y),\quad p' = \Pr_{o\sim\mu}(A(o)=y')$$

We use Neyman–Pearson Lemma to check whether Eq 13 holds by computing a lower bound lb of the LHS and an upper bound ub of the RHS by solving the following optimization problems,

$$\begin{aligned}
&\text{lb} \triangleq \min_{A^? \in\mathcal{A}}\Pr_{o\sim\widetilde{\mu}}(A^?(o)=y^*),\ \text{ub} \triangleq \max_{A^?\in\mathcal{A}}\Pr_{o\sim\widetilde{\mu}}(A^?(o)=y'),\\
&s.t.\ \Pr_{o\sim\mu}(A^?(o)=y^*)=p^*,\ \Pr_{o\sim\mu}(A^?(o)=y')=p'\\
&\widetilde{D}\in S_r^{\pi}(D), \widetilde{\mathbf{x}}\in\pi(\mathbf{x},y)_1,
\end{aligned} \tag{14}$$

Theorem B.1 (Neyman–Pearson Lemma for FL$_s$, F$_s$, L in the Multi-class Case)**.** *Let $\widetilde{D}$ and $\widetilde{\mathbf{x}}$ be a maximally perturbed dataset and test input, i.e., $|\widetilde{D}\backslash D| = r$, $\|\widetilde{\mathbf{x}}-\mathbf{x}\|_0 = s$, and $\|\widetilde{\mathbf{x}}_i-\mathbf{x}_i\|_0+\mathbb{1}_{\widetilde{y}_i\neq y_i} = s$, for each perturbed example $(\widetilde{\mathbf{x}}_i,\widetilde{y}_i)$ in $\widetilde{D}$. Let $i_{\text{lb}} \triangleq \text{argmin}_{i\in[1,m]}\sum_{j=1}^{i}p_\mu(\mathcal{L}_j) \geq p^*$ and*

$i_{\mathrm{ub}} \triangleq \mathrm{argmax}_{i\in[1,m]} \sum_{j=i}^{m} p_\mu(\mathcal{L}_j) \geq p'$, *The algorithm* $\bar{A}^?$ *is the minimizer of Eq 14 and its behaviors among* $\forall i \in \{1,\dots,m\}. \forall o \in \mathcal{L}_i$ *are specified as*

$$\Pr(A^?(o) = y^*) = \begin{cases} 1, & i < i_{\mathrm{lb}} \\ \frac{p^* - \sum_{j=1}^{i-1} p_\mu(\mathcal{L}_j)}{p_\mu(\mathcal{L}_i)}, & i = i_{\mathrm{lb}} \\ 0, & i > i_{\mathrm{lb}} \end{cases} . \Pr(A^?(o) = y') = \begin{cases} 0, & i < i_{\mathrm{ub}} \\ \frac{p' - \sum_{j=i+1}^{m} p_\mu(\mathcal{L}_j)}{p_\mu(\mathcal{L}_i)}, & i = i_{\mathrm{ub}} \\ 1, & i > i_{\mathrm{ub}} \end{cases}$$

Then,

$$\mathrm{lb} = \sum_{j=1}^{i_{\mathrm{lb}}-1} p_{\widetilde{\mu}}(\mathcal{L}_j) + \left(p^* - \sum_{j=1}^{i_{\mathrm{lb}}-1} p_\mu(\mathcal{L}_j)\right) / \eta_{i_{\mathrm{lb}}}, \mathrm{ub} = \sum_{j=i_{\mathrm{ub}}+1}^{m} p_{\widetilde{\mu}}(\mathcal{L}_j) + \left(p' - \sum_{j=i_{\mathrm{ub}}+1}^{m} p_\mu(\mathcal{L}_j)\right) / \eta_{i_{\mathrm{ub}}}$$

Theorem B.1 is a direct application of Neyman–Pearson Lemma. And Lemma 4 in Lee et al. [18] proves the maximally perturbed dataset and test input achieve the worst-case bound lb and ub.

B.1 Computing the Certified Radius of BagFlip for Perturbations FL_s and L

Computing the Certified Radius for FL_s In Eq 7, we need to consider the flipping of labels,

$$\underbrace{\left(\sum_{i=1}^{k} \|\mathbf{x}'_{w_i} - \mathbf{x}_{w_i}\|_0 + \mathbb{1}_{y'_{w_i} \neq y_{w_i}} + \|\mathbf{x}' - \mathbf{x}\|_0\right)}_{\Delta} - \underbrace{\left(\sum_{i=1}^{k} \|\mathbf{x}'_{w_i} - \widetilde{\mathbf{x}}_{w_i}\|_0 + \mathbb{1}_{y'_{w_i} \neq \widetilde{y}_{w_i}} + \|\mathbf{x}' - \widetilde{\mathbf{x}}\|_0\right)}_{\widetilde{\Delta}} = t$$

In Eq 8, the definition of $T(c, t)$ should be modified to,

$$\sum_{\substack{0\leq u_1,\dots,u_c \leq d+1 \\ 0 \leq u_0 \leq d}} \sum_{\substack{0\leq v_1,\dots,v_c \leq d+1 \\ 0 \leq v_0 \leq d \\ u_0 - v_0 + \dots + u_c - v_c = t}} \prod_{i=1}^{c} L(u_i, v_i; s, d+1) b^{u_i} a^{d+1-u_i} L(u_0, v_0; s, d) b^{u_0} a^{d-u_0}$$

The algorithm associated with Theorem 5.4 should be modified as

$$T(0, t) = \sum_{u=\max(0,t)}^{\min(d,t+d)} L(u, u-t; s, d) b^u a^{d-u}, \ \forall -d \leq t \leq d,$$

$$T(c, t) = \sum_{t_1=\max(-d,t-cd-c+1)}^{\min(d,t+cd+c-1)} T(c-1, t-t_1) G(t_1), \ \forall c > 0, -(c+1)d - c \leq t \leq (c+1)d + c,$$

where $G(t)$ is defined as

$$G(t) = \sum_{u=\max(0,t)}^{\min(d+1,t+d+1)} L(u, u-t; s, d+1) b^u a^{d+1-u}, \ \forall -d-1 \leq t \leq d+1$$

Computing the Certified Radius for L In Eq 7, we only need to consider the flipping of labels,

$$\underbrace{\left(\sum_{i=1}^{k} \mathbb{1}_{y'_{w_i} \neq y_{w_i}}\right)}_{\Delta} - \underbrace{\left(\sum_{i=1}^{k} \mathbb{1}_{y'_{w_i} \neq \widetilde{y}_{w_i}}\right)}_{\widetilde{\Delta}} = t$$

In the rest of the computation of $T(c, t)$, we set $d = 1$, i.e., consider the label as a one-dimension feature.

B.2 Practical Perspectives

Estimation of p^* and p'. For each test example $\mathbf{x}$, we need to estimate p^* and p' of the smoothed algorithm $\bar{A}$ given the benign dataset D. We use Monte Carlo sampling to compute p^* and p'. Specifically, for each test input $\mathbf{x}$, we train N algorithms with the datasets $\dot{D}_1, \dots, \dot{D}_N$ and evaluate these algorithms on input $\dot{\mathbf{x}}_i$, where $(\dot{D}_i, \dot{\mathbf{x}}_i)$ is sampled from distribution $\mu(D, \mathbf{x})$. We count the predictions equal to label y as $N_y = \sum_{i=1}^{N} \mathbb{1}_{A(\dot{D}_i, \dot{\mathbf{x}}_i)=y}$ and use the Clopper-Pearson interval to estimate p^* and p'.

$$p^* = \mathrm{Beta}(\frac{\alpha}{|\mathcal{C}|}; N_{y^*}, N - N_{y^*} + 1), \quad p' = \mathrm{Beta}(\frac{1-\alpha}{|\mathcal{C}|}; N_{y'} + 1, N - N_{y'})$$

where $1 - \alpha$ is the confidence level, $|\mathcal{C}|$ is the number of different labels, and and $\mathrm{Beta}(\beta; \lambda, \theta)$ is the βth quantile of the Beta distribution of parameter λ and θ. We further tighten the estimation of p' by $\min(p', 1 - p^*)$ because $p^* + p' \leq 1$ by definition.

However, it is computationally expensive to retrain N algorithms for each test input. We can reuse the trained N algorithms to estimate p^* of m test inputs with a simultaneous confidence level at least $1 - \alpha$ by using *Bonferroni correction*. Specifically, we evenly divide α to $\frac{\alpha}{m}$ when estimating for each test input. In the evaluation, we set m to be the size of the test set.

Computation of the certified radius. Previous sections have introduced how to check whether Eq 4 holds for a specific r. We can use binary search to find the certified radius given the estimated p^* and p'. Although checking Eq 4 has been reduced to polynomial time, it might be infeasible for in-time prediction in the real scenario. We propose to memoize the certified radius by enumerating all possibilities of pairs of p^* and p' beforehand so that checking Eq 4 can be done in $O(1)$. Notice that a pair of p^* and p' is determined by N_{y^*} and $N_{y'}$. Recall that N is the number of trained algorithms. N_{y^*} and $N_{y'}$ can have $O(N^2)$ different pairs ($O(N)$ in the binary-classification case).

B.3 A Relaxation of the Neyman–Pearson Lemma

We introduce a relaxation of the Neyman–Pearson lemma for the multi-class case.

Theorem B.2 (Relaxation of the Lemma). *Define* lb *for the original subspaces* $\mathcal{L}_1, \dots, \mathcal{L}_m$ *as in Theorem B.1. Define* lb_δ *for* $\{\mathcal{L}_i\}_{i \in B}$ *as in Theorem B.1 by underapproximating* p^* *as* $p^* - \sum_{i \notin B} p_\mu(\mathcal{L}_i)$*. Define* ub *for the original subspaces* $\mathcal{L}_1, \dots, \mathcal{L}_m$ *as in Theorem B.1. Define* ub' *for underapproximated subspaces* $\{\mathcal{L}_i\}_{i \in B}$ *as in Theorem B.1 with* p' *and let* $\mathrm{ub}_\delta = \mathrm{ub}' + \sum_{i \notin B} p_{\widetilde{\mu}}(\mathcal{L}_i)$*. Then, we have* ***Soundness:*** $\mathrm{lb}_\delta \leq \mathrm{lb}, \mathrm{ub}_\delta \geq \mathrm{ub}$ *and* δ***-Tightness:*** *Let* $\delta \triangleq \sum_{i \notin B} p_{\widetilde{\mu}}(\mathcal{L}_i)$*, then* $\mathrm{lb}_\delta + \delta \geq \mathrm{lb}, \mathrm{ub}_\delta - \delta \geq \mathrm{ub}$.

C Proofs

Proof of Theorem 5.2.

Proof. If we know an outcome $o \in \mathcal{L}_{c,t}$ has Δ distance from the clean data, then o has $(k+1)d - \Delta$ features unchanged and Δ features changed when sampling from μ. Thus, according to the definition of PMF in Eq 10, $p_\mu(o) = \frac{1}{n^k}\rho^{(k+1)d-\Delta}\gamma^\Delta$ for all $o \in \mathcal{L}_{c,t}$. Similarly, $p_{\widetilde{\mu}}(o) = \frac{1}{n^k}\rho^{(k+1)d-\widetilde{\Delta}}\gamma^{\widetilde{\Delta}}$ for all $o \in \mathcal{L}_{c,t}$. By the definition of likelihood ratios, we have $\forall o \in \mathcal{L}_{c,t}.\ \eta(o) = \eta_{c,t} = p_\mu(o)/p_{\widetilde{\mu}}(o) = \gamma^{\Delta - \widetilde{\Delta}}\rho^{\widetilde{\Delta} - \Delta} = \left(\frac{\gamma}{\rho}\right)^t$. □

Proof of Theorem 5.3.

Proof. We first define a subset of $\mathcal{L}_{c,t}$ as $\mathcal{L}_{c,t,\Delta}$,

$$\begin{aligned}\mathcal{L}_{c,t,\Delta} = \{&((\{(\mathbf{x}'_{w_i}, y_{w_i})\}_i, \mathbf{x}') \mid \\ &\sum_{i=1}^{k} \mathbb{1}_{\mathbf{x}_{w_i} \neq \widetilde{\mathbf{x}}_{w_i}} = c, \\ &\left(\sum_{i=1}^{k} \|\mathbf{x}'_{w_i} - \mathbf{x}_{w_i}\|_0 + \|\mathbf{x}' - \mathbf{x}\|_0\right) - \left(\sum_{i=1}^{k} \|\mathbf{x}'_{w_i} - \widetilde{\mathbf{x}}_{w_i}\|_0 + \|\mathbf{x}' - \widetilde{\mathbf{x}}\|_0\right) = t, \\ &\left(\sum_{i=1}^{k} \|\mathbf{x}'_{w_i} - \mathbf{x}_{w_i}\|_0 + \|\mathbf{x}' - \mathbf{x}\|_0\right) = \Delta\}\end{aligned}$$

Denote the size of $\mathcal{L}_{c,t,\Delta}$ as $|\mathcal{L}_{c,t,\Delta}|$, then $p_\mu(\mathcal{L}_{c,t})$ can be computed as

$$p_\mu(\mathcal{L}_{c,t}) = \sum_{0\le\Delta\le(c+1)d} p_\mu(\mathcal{L}_{c,t,\Delta}) = \sum_{0\le\Delta\le(c+1)d} \frac{1}{n^k}\gamma^\Delta\rho^{d-\Delta}|\mathcal{L}_{c,t,\Delta}| \tag{15}$$

Because every outcome in $\mathcal{L}_{c,t,\Delta}$ has the same probability mass and we only need to count the size of $\mathcal{L}_{c,t,\Delta}$.

$$|\mathcal{L}_{c,t,\Delta}| = \binom{k}{c} r^c (n-r)^{k-c} \sum_{\substack{0\le\widetilde{\Delta}_0,\ldots,\widetilde{\Delta}_c\le d\\ 0\le\Delta_0,\ldots,\Delta_c\le d\\ \Delta_0-\widetilde{\Delta}_0+\ldots+\Delta_c-\widetilde{\Delta}_c=t\\ \Delta_0+\ldots+\Delta_c=\Delta}} \prod_{i=0}^{c} L(\Delta_i,\widetilde{\Delta}_i;s,d), \tag{16}$$

where $L(\Delta,\widetilde{\Delta};s,d)$ is defined as, similarly in the Lemma 5 of Lee et al. [18],

$$\sum_{i=\max(0,\widetilde{\Delta}-s)}^{\min(\Delta,d-s,\lfloor\frac{\Delta+\widetilde{\Delta}-s}{2}\rfloor)} (K-1)^j \binom{s}{j}\binom{s-j}{\Delta-i-j} K^i \binom{d-s}{i},$$

where $j = \Delta + \widetilde{\Delta} - 2i - s$.

The binomial term in Eq 16 represents the different choices of selecting the k indices with c perturbed indices from a pool containing r perturbed indices and $n-r$ clean indices. The rest of Eq 16 counts the number of different choices of flips. Specifically, we enumerate Δ and $\widetilde{\Delta}$ as $\sum_{i=0}^{c}\Delta_i$ and $\sum_{i=0}^{c}\widetilde{\Delta}_i$, where $i=0$ denotes the test input. And $L(\Delta_i,\widetilde{\Delta}_i;s,d)$ counts the number of different choices of flips for each example (see Lemma 5 of Lee et al. [18] for the derivation of $L(\Delta_i,\widetilde{\Delta}_i;s,d)$).

Then, plug Eq 16 into Eq 15, we have

$$\begin{aligned}
p_\mu(\mathcal{L}_{c,t}) &= \sum_{0\le\Delta\le(c+1)d} \frac{1}{n^k}\gamma^\Delta\rho^{d-\Delta}\binom{k}{c} r^c(n-r)^{k-c} \sum_{\substack{0\le\widetilde{\Delta}_0,\ldots,\widetilde{\Delta}_c\le d\\ 0\le\Delta_0,\ldots,\Delta_c\le d\\ \Delta_0-\widetilde{\Delta}_0+\ldots+\Delta_c-\widetilde{\Delta}_c=t\\ \Delta_0+\ldots+\Delta_c=\Delta}} \prod_{i=0}^{c} L(\Delta_i,\widetilde{\Delta}_i;s,d)\\
&= \sum_{0\le\Delta\le(c+1)d} \frac{1}{n^k}\binom{k}{c} r^c(n-r)^{k-c} \sum_{\substack{0\le\widetilde{\Delta}_0,\ldots,\widetilde{\Delta}_c\le d\\ 0\le\Delta_0,\ldots,\Delta_c\le d\\ \Delta_0-\widetilde{\Delta}_0+\ldots+\Delta_c-\widetilde{\Delta}_c=t\\ \Delta_0+\ldots+\Delta_c=\Delta}} \prod_{i=0}^{c} L(\Delta_i,\widetilde{\Delta}_i;s,d)\gamma^{\Delta_i}\rho^{d-\Delta_i}\\
&=\frac{1}{n^k}\binom{k}{c} r^c(n-r)^{k-c} \sum_{\substack{0\le\widetilde{\Delta}_0,\ldots,\widetilde{\Delta}_c\le d\\ 0\le\Delta_0,\ldots,\Delta_c\le d\\ \Delta_0-\widetilde{\Delta}_0+\ldots+\Delta_c-\widetilde{\Delta}_c=t}} \prod_{i=0}^{c} L(\Delta_i,\widetilde{\Delta}_i;s,d)\gamma^{\Delta_i}\rho^{d-\Delta_i}\\
&=\text{Binom}(c;k,\frac{r}{n}) \sum_{\substack{0\le\widetilde{\Delta}_0,\ldots,\widetilde{\Delta}_c\le d\\ 0\le\Delta_0,\ldots,\Delta_c\le d\\ \Delta_0-\widetilde{\Delta}_0+\ldots+\Delta_c-\widetilde{\Delta}_c=t}} \prod_{i=0}^{c} L(\Delta_i,\widetilde{\Delta}_i;s,d)\gamma^{\Delta_i}\rho^{d-\Delta_i}
\end{aligned}$$

$\square$

Proof of Theorem 5.4.

Proof. From Eq 8 in Theorem 5.3, then for $c \geq 1$, we have

$$
\begin{aligned}
T(c,t) &= \sum_{0 \leq \Delta_0, \ldots, \Delta_c \leq d} \; \sum_{\substack{0 \leq \widetilde{\Delta}_0, \ldots, \widetilde{\Delta}_c \leq d \\ \Delta_0 - \widetilde{\Delta}_0 + \ldots + \Delta_c - \widetilde{\Delta}_c = t}} \prod_{i=0}^{c} L(\Delta_i, \widetilde{\Delta}_i; s, d) \gamma^{\Delta_i} \rho^{d-\Delta_i} \\
&= \sum_{-d \leq t_1 \leq d} \sum_{0 \leq \Delta_c \leq d} \sum_{\substack{0 \leq \widetilde{\Delta}_c \leq d \\ \Delta_c - \widetilde{\Delta}_c = t_1}} L(\Delta_c, \widetilde{\Delta}_c; s, d) \gamma^{\Delta_c} \rho^{d-\Delta_c} \times \\
&\quad \sum_{0 \leq \Delta_0, \ldots, \Delta_{c-1} \leq d} \; \sum_{\substack{0 \leq \widetilde{\Delta}_0, \ldots, \widetilde{\Delta}_{c-1} \leq d \\ \Delta_0 - \widetilde{\Delta}_0 + \ldots + \Delta_{c-1} - \widetilde{\Delta}_{c-1} = t - t_1}} \prod_{i=0}^{c-1} L(\Delta_i, \widetilde{\Delta}_i; s, d) \gamma^{\Delta_i} \rho^{d-\Delta_i} \\
&= \sum_{-d \leq t_1 \leq d} T(0, t_1) T(c-1, t-t_1)
\end{aligned}
$$

□

Proof of Theorem 6.1.

Before giving the formal proof, we motivate the proof by the following knapsack problem, where each item can be divided arbitrarily. This allows a greedy algorithm to solve the problem, the same as the greedy process in Theorem 5.1.

Example C.1. *Suppose we have m items with volume $p_\mu(\mathcal{L}_i)$ and cost $p_{\widetilde{\mu}}(\mathcal{L}_i)$. We have a knapsack with volume p^*. Determine the best strategy to fill the knapsack with the minimal cost* lb. *Note that each item can be divided arbitrarily.*

The greedy algorithm sorts the item descendingly by "volume per cost" $p_\mu(\mathcal{L}_i)/p_{\widetilde{\mu}}(\mathcal{L}_i)$ (likelihood ratio) and select items until the knapsack is full. The last selected item $\mathcal{L}_{i_{\mathrm{lb}}}$ will be divided to fill the knapsack. Define the best solution as S in this case and the minimal cost as lb.

Now consider Theorem 6.1, which removes items $\{\mathcal{L}_i\}_{i \notin B}$ and reduces the volume of knapsack by the sum of the removed items' volume. Applying the greedy algorithm again, denote the best solution as S' and the minimal cost in this case as lb_δ.

Soundness: The above process of removing items and reducing volume of knapsack is equivalent to just setting the cost of items in $\{\mathcal{L}_i\}_{i \notin B}$ as zero. Then, the new cost lb_δ will be better than before (less than lb) because the cost of some items has been set to zero.

δ-tightness: If we put removed items back to the reduced knapsack solution S', this new solution is a valid selection in the original problem with cost $\mathrm{lb}_\delta + \sum_{i \notin B} p_{\widetilde{\mu}}(\mathcal{L}_i)$, and this cost cannot be less than the minimal cost lb, i.e., $\mathrm{lb}_\delta + \sum_{i \notin B} p_{\widetilde{\mu}}(\mathcal{L}_i) \geq \mathrm{lb}$.

Proof. We first consider a base case when $|B| = m - 1$, i.e., only one subspace is underapproximated. We denote the index of that subspace as i'. Consider i_{lb} computed in Theorem 5.1.

- If $i_{\mathrm{lb}} > i'$, then the likelihood ratio from $\{\mathcal{L}_i\}_{i \notin B}$ is used for computing lb in Theorem 5.1, meaning $\mathrm{lb}_\delta = \mathrm{lb} - p_{\widetilde{\mu}}(\mathcal{L}_{i'})$. This implies both soundness and δ-tightness.
- If $i_{\mathrm{lb}} = i'$, then $\mathcal{L}_{i'}$ partially contributes to the computation of lb in Theorem 5.1. First, $\mathrm{lb}_\delta \leq \mathrm{lb}$ because the computation of lb_δ can only sum up to $i_{\mathrm{lb}} - 1$ items (it cannot sum i_{lb}th item), which implies $\mathrm{lb} \geq \sum_{i=1}^{i_{\mathrm{lb}}-1} p_{\widetilde{\mu}}(\mathcal{L}_i) \geq \mathrm{lb}_\delta$.

  Next, we are going to prove $\mathrm{lb}_\delta + p_{\widetilde{\mu}}(\mathcal{L}_{i'}) \geq \mathrm{lb}$. Suppose the additional budget $p_\mu(\mathcal{L}_{i'})$ for lb selects additional subspaces (than underapproximated lb_δ) with likelihood ratio $\frac{p_\mu(\mathcal{L}_{i'-q})}{p_{\widetilde{\mu}}(\mathcal{L}_{i'-q})}, \ldots, \frac{p_\mu(\mathcal{L}_{i'-1})}{p_{\widetilde{\mu}}(\mathcal{L}_{i'-1})}, \frac{p_\mu(l_{i'})}{p_{\widetilde{\mu}}(l_{i'})}$ such that $p_\mu(\mathcal{L}_{i'}) = \sum_{j=1}^{q} p_\mu(\mathcal{L}_{i'-j}) + p_\mu(l_{i'})$, $\mathrm{lb} = \mathrm{lb}_\delta + \sum_{j=1}^{q} p_{\widetilde{\mu}}(\mathcal{L}_{i'-j}) + p_{\widetilde{\mu}}(l_{i'})$, and $l_{i'} \subseteq \mathcal{L}_{i'}$. Then we have $\mathrm{lb} \leq \mathrm{lb}_\delta + p_{\widetilde{\mu}}(\mathcal{L}_{i'})$ because $\frac{p_\mu(\mathcal{L}_{i'-q})}{p_{\widetilde{\mu}}(\mathcal{L}_{i'-q})} \geq \ldots \geq \frac{p_\mu(\mathcal{L}_{i'-1})}{p_{\widetilde{\mu}}(\mathcal{L}_{i'-1})} \geq \frac{p_\mu(l_{i'})}{p_{\widetilde{\mu}}(l_{i'})} = \frac{p_\mu(\mathcal{L}_{i'})}{p_{\widetilde{\mu}}(\mathcal{L}_{i'})}$ implies $\sum_{j=1}^{q} p_{\widetilde{\mu}}(\mathcal{L}_{i'-j}) + p_{\widetilde{\mu}}(l_{i'}) \leq$

$p_{\widetilde{\mu}}(\mathcal{L}_{i'})$. To see this implication, we have $p_{\widetilde{\mu}}(\mathcal{L}_{i'}) = p_\mu(\mathcal{L}_{i'})/\eta_{i'} = \sum_{j=1}^{q} p_\mu(\mathcal{L}_{i'-j})/\eta_{i'} + p_\mu(l_{i'})/\eta_{i'} \geq \sum_{j=1}^{q} p_\mu(\mathcal{L}_{i'-j})/\eta_{i'-j} + p_\mu(l_{i'})/\eta_{i'} = \sum_{j=1}^{q} p_{\widetilde{\mu}}(\mathcal{L}_{i'-j}) + p_{\widetilde{\mu}}(l_{i'})$.

- If $i_{\mathrm{lb}} < i'$, then $\mathrm{lb}_\delta \leq \mathrm{lb}$ because lb has more budget as p^* than $p^* - p_\mu(\mathcal{L}_{i'})$. Suppose the additional budget $p_\mu(\mathcal{L}_{i'})$ for lb selects additional subspaces (than underapproximated lb_δ) with likelihood ratio $\frac{p_\mu(\mathcal{L}_{i_{\mathrm{lb}}-q})}{p_{\widetilde{\mu}}(\mathcal{L}_{i_{\mathrm{lb}}-q})}, \ldots, \frac{p_\mu(\mathcal{L}_{i_{\mathrm{lb}}})}{p_{\widetilde{\mu}}(\mathcal{L}_{i_{\mathrm{lb}}})}$ (we assume it selects the whole $\mathcal{L}_{i_{\mathrm{lb}}}$ for simplicity, and if it selects a subset of $\mathcal{L}_{i_{\mathrm{lb}}}$ can be proved similarly) such that $p_\mu(\mathcal{L}_{i'}) = \sum_{j=0}^{q} p_\mu(\mathcal{L}_{i_{\mathrm{lb}}-j})$ and $\mathrm{lb} = \mathrm{lb}_\delta + \sum_{j=0}^{q} p_{\widetilde{\mu}}(\mathcal{L}_{i_{\mathrm{lb}}-j})$. Then we have $\mathrm{lb} \leq \mathrm{lb}_\delta + p_{\widetilde{\mu}}(\mathcal{L}_{i'})$ because $\frac{p_\mu(\mathcal{L}_{i_{\mathrm{lb}}-j})}{p_{\widetilde{\mu}}(\mathcal{L}_{i_{\mathrm{lb}}-j})} \geq \ldots \geq \frac{p_\mu(\mathcal{L}_{i_{\mathrm{lb}}})}{p_{\widetilde{\mu}}(\mathcal{L}_{i_{\mathrm{lb}}})} \geq \frac{p_\mu(\mathcal{L}_{i'})}{p_{\widetilde{\mu}}(\mathcal{L}_{i'})}$ implies $\sum_{j=0}^{q} p_{\widetilde{\mu}}(\mathcal{L}_{i_{\mathrm{lb}}-j}) \leq p_{\widetilde{\mu}}(\mathcal{L}_{i'})$. The reason of implication can be proved in a similar way as above.

We then consider $|B| < m - 1$, i.e., more than one subspaces are underapproximated. We separate $\{\mathcal{L}_i\}_{i \notin B}$ into two parts, one $\{\mathcal{L}_{i'}\}$ contains any one of the subspaces, the other contains the rest $\{\mathcal{L}_i\}_{i \notin B \wedge i \neq i'}$.

Denote lb'_δ for $\{\mathcal{L}_i\}_{i \in B \vee i = i'}$ as in Theorem 5.1 by underapproximating p^* as $p^* - \sum_{i \notin B} p_\mu(\mathcal{L}_i) + p_\mu(\mathcal{L}_{i'})$. By inductive hypothesis, $\mathrm{lb} \geq \mathrm{lb}'_\delta$ and $\mathrm{lb} \leq \mathrm{lb}'_\delta + \sum_{i \notin B} p_\mu(\mathcal{L}_i) - p_\mu(\mathcal{L}_{i'})$. By the same process of the above proof when one subspace is underapproximated (comparing $\{\mathcal{L}_i\}_{i \in B \vee i = i'}$ with $\{\mathcal{L}_i\}_{i \in B}$), we have $\mathrm{lb}'_\delta \geq \mathrm{lb}_\delta$ and $\mathrm{lb}'_\delta \leq \mathrm{lb}_\delta + p_\mu(\mathcal{L}_{i'})$. Combining the results above, we have $\mathrm{lb} \geq \mathrm{lb}_\delta$ and $\mathrm{lb} \leq \mathrm{lb}_\delta + \sum_{i \notin B} p_\mu(\mathcal{L}_i)$.

□

D A KL-divergence Bound on the Certified Radius

We can use KL divergence [8] to get a looser but computationally-cheaper bound on the certified radius. Here, we certify the trigger-less case for F_s.

Theorem D.1. *Consider the binary classification case, Eq 4 holds if*

$$r < \frac{n \log(4p^*(1-p^*))}{2k \log(\frac{\gamma}{\rho})(\rho - \gamma)s}, \tag{17}$$

where n is the size of the training set, r is the certified radius, s is the number of the perturbed features, k is the size of each bag, and ρ, γ are the probabilities of a featuring remaining the same and being flipped.

Lemma D.1. *Define T as*

$$T = \sum_{u=0}^{d} \sum_{v=0}^{d} L(u, v; s, d) \gamma^u \rho^{d-u} (v - u) \tag{18}$$

where $L(u, v; s, d)$ is the same quantity defined in Lee et al. [18]. Then, we have

$$T = (\rho - \gamma)s$$

Proof of the Theorem D.1.

Proof. Denote $D' \sim \mu(D)$ and $D'' \sim \mu(\widetilde{D})$, from the theorem in Example 5 of [28], Eq 4 holds if

$$\mathrm{KL}(D'' \| D') < -\frac{1}{2} \log(4p^*(1-p^*)) \tag{19}$$

Denote the PMF of selecting an index w and flip $\mathbf{x}_w$ to $\mathbf{x}'_w$ by $\mu(D)$ as $p_\mu(\mathbf{x}'_w) = \frac{\rho^d}{n}\left(\frac{\gamma}{\rho}\right)^{\|\mathbf{x}'_w - \mathbf{x}_w\|_0}$. Similarly, the PMF of selecting an index w and flip $\mathbf{x}_w$ to $\mathbf{x}'_w$ by $\mu(\widetilde{D})$ as $p_{\widetilde{\mu}}(\mathbf{x}'_w) = \frac{\rho^d}{n}\left(\frac{\gamma}{\rho}\right)^{\|\mathbf{x}'_w - \widetilde{\mathbf{x}}_w\|_0}$. We now calculate the KL divergence between the distribution generated from the perturbed dataset $\widetilde{D}$ and the distribution generated from the original dataset D.

$$\mathrm{KL}(D''\|D')$$

$$=k\mathrm{KL}(D_1''\|D_1') \tag{20}$$

$$=k\sum_{w=1}^{|D|}\sum_{\mathbf{x}_w'\in[K]^d} p_{\widetilde{\mu}}(\mathbf{x}_w')\log\frac{p_{\widetilde{\mu}}(\mathbf{x}_w')}{p_{\mu}(\mathbf{x}_w')}$$

$$=k\sum_{\mathbf{x}_w\neq\widetilde{\mathbf{x}}_w}\sum_{\mathbf{x}_w'\in[K]^d} p_{\widetilde{\mu}}(\mathbf{x}_w')\log\frac{p_{\widetilde{\mu}}(\mathbf{x}_w')}{p_{\mu}(\mathbf{x}_w')} \tag{21}$$

$$\leq kr\sum_{\mathbf{x}_w'\in[K]^d}\frac{\rho^d}{n}\left(\frac{\gamma}{\rho}\right)^{\|\mathbf{x}_w'-\widetilde{\mathbf{x}}_w\|_0}\log(\frac{\gamma}{\rho})(\|\mathbf{x}_w'-\widetilde{\mathbf{x}}_w\|_0-\|\mathbf{x}_w'-\mathbf{x}_w\|_0)$$

$$=k\frac{r}{n}\rho^d\log(\frac{\gamma}{\rho})\sum_{\mathbf{x}_w'\in[K]^d}\left(\frac{\gamma}{\rho}\right)^{\|\mathbf{x}_w'-\widetilde{\mathbf{x}}_w\|_0}(\|\mathbf{x}_w'-\widetilde{\mathbf{x}}_w\|_0-\|\mathbf{x}_w'-\mathbf{x}_w\|_0) \tag{22}$$

where $\mathrm{KL}(D_1''\|D_1')$ is the KL divergence of the first selected instance. We have Eq 20 because each selected instance is independent. We have Eq 21 because the $p_{\mu}(\mathbf{x}_w')$ and $p_{\widetilde{\mu}}(\mathbf{x}_w')$ only differs when the wth instance is perturbed.

The attacker can modify $\widetilde{D}$ to maximize Eq 22. And the Lemma 4 in Lee et al. [18] states that the maximal value is achieved when $\widetilde{\mathbf{x}}_w$ has exact s features flipped to another value. Now suppose there are s features flipped in $\widetilde{\mathbf{x}}_w$, we then need to compute Eq 22. If we denote $\|\mathbf{x}_w'-\widetilde{\mathbf{x}}_w\|_0$ as u and $\|\mathbf{x}_w'-\mathbf{x}_w\|_0$ as v, and we count the size of the set $L(u,v;s,d)=\{\mathbf{x}_w'\in[K]^d \mid \|\mathbf{x}_w'-\widetilde{\mathbf{x}}_w\|_0=u, \|\mathbf{x}_w'-\mathbf{x}_w\|_0=v, \|\mathbf{x}_w-\widetilde{\mathbf{x}}_w\|_0=s\}$, then we can compute Eq 22 as

$$k\frac{r}{n}\rho^d\log(\frac{\rho}{\gamma})\sum_{u=0}^{d}\sum_{v=0}^{d}|L(u,v;s,d)|\left(\frac{\gamma}{\rho}\right)^u(v-u)$$

Let T be defined as in Eq 18, we then have

$$\mathrm{KL}(D''\|D')=k\frac{r}{n}\log(\frac{\rho}{\gamma})T \tag{23}$$

Combine Eq 19, Eq 23, and Lemma D.1, we have

$$k\frac{r}{n}\log(\frac{\rho}{\gamma})(\rho-\gamma)s\leq\mathrm{KL}(D''\|D')<-\frac{1}{2}\log(4p^*(1-p^*))$$

$$r<\frac{n\log(4p^*(1-p^*))}{2k\log(\frac{\gamma}{\rho})(\rho-\gamma)s}$$

□

Proof of the Lemma D.1.

Proof. Notice that the value of T does not defend on the feature dimension d. Thus, we prove the lemma by induction on d. We further denote the value of T under the feature dimension d as T_d.

Let $L(u,v;s,d)$ be defined as in the Lemma 5 of Lee et al. [18],

$$\sum_{i=\max(0,v-s)}^{\min(u,d-s,\lfloor\frac{u+v-s}{2}\rfloor)}(K-1)^j\binom{s}{j}\binom{s-j}{u-i-j}K^i\binom{d-s}{i},$$

where $j=u+v-2i-s$.

Base case. Because $0\leq s\leq d$, when $d=0$, it is easy to see $T_0=s(\rho-\gamma)=0$.

Induction case. We first prove $T_{d+1}=s(\rho-\gamma)$ under a special case, where $s=d+1$, given the inductive hypothesis $T_d=s(\rho-\gamma)$ for $s=d$.

By definition

$$T_{d+1} = \sum_{u=0}^{d+1} \sum_{v=0}^{d+1} |L(u, v; s, d+1)| \gamma^u \rho^{d+1-u} (v - u), \tag{24}$$

By the definition of $L(u, v; s, d)$, we have the following equation when $0 \leq s \leq d$,

$$|L(u, v; d+1, d+1)| = (K-1)|L(u-1, v-1; d, d)| + |L(u-1, v; d, d)| + |L(u, v-1; d, d)| \tag{25}$$

and

$$\sum_{u=0}^{d} \sum_{v=0}^{d} |L(u, v; d, d)| \gamma^u \rho^{d-u} = 1 \tag{26}$$

Plug Eq 25 into Eq 24, we have the following equations when $s = d+1$,

$$\begin{align}
T_{d+1} =& \sum_{u=0}^{d+1} \sum_{v=0}^{d+1} (K-1)|L(u-1, v-1; d, d)| \gamma^u \rho^{d+1-u} (v-u) + \nonumber \\
& \sum_{u=0}^{d+1} \sum_{v=0}^{d+1} |L(u-1, v; d, d)| \gamma^u \rho^{d+1-u} (v-u) + \nonumber \\
& \sum_{u=0}^{d+1} \sum_{v=0}^{d+1} |L(u, v-1; d, d)| \gamma^u \rho^{d+1-u} (v-u) \nonumber \\
=& \gamma(K-1) \sum_{u=0}^{d} \sum_{v=0}^{d} |L(u, v; d, d)| \gamma^u \rho^{d-u} (v-u) + \tag{27} \\
& \gamma \sum_{u=0}^{d} \sum_{v=0}^{d} |L(u, v; d, d)| \gamma^u \rho^{d-u} (v-u-1) + \tag{28} \\
& \rho \sum_{u=0}^{d} \sum_{v=0}^{d} |L(u, v; d, d)| \gamma^u a^{d-u} (v-u+1) \tag{29} \\
=& \sum_{u=0}^{d} \sum_{v=0}^{d} |L(u, v; d, d)| \gamma^u \rho^{d-u} [(v-u) + (\rho - \gamma)] \tag{30} \\
=& d(\rho - \gamma) + (\rho - \gamma) \tag{31} \\
=& s(\rho - \gamma) \nonumber
\end{align}$$

We have Eq 25, Eq 27 and Eq 28 because $L(u, v; s, d) = 0$ when $u > d$, $v > d$, $u < 0$, or $v < 0$. We have Eq 30 because $\gamma(K-1) + \gamma + \rho = 1$ as γ is defined as $\frac{1-\rho}{K}$. We have Eq 31 by plugging in Eq 26.

Next, we are going to show that $T_{d+1} = s(\rho - \gamma)$ for all $0 \leq s \leq d$, given the inductive hypothesis $T_d = s(\rho - \gamma)$ for all $0 \leq s \leq d$.

By the definition of $L(u, v; s, d)$, we have the following equations when $0 \leq s \leq d$,

$$|L(u, v; s, d+1)| = |L(u, v; s, d)| + K|L(u-1, v-1; s, d)| \tag{32}$$

Table 3: This paper compared to other approaches. RAB [39] can handle perturbation F^*_s that perturbs the input within a l_2-norm ball of radius s.

Approach	Perturbation π	Probability measure μ	Goal
Jia et al. [15], Chen et al. [4]	Any	Bagging	Trigger-less
Rosenfeld et al. [28]	L	Noise	Trigger-less
RAB [39]	F^*_s	Noise	Trigger-less, Backdoor
Wang et al. [35]	$\mathrm{FL}_s, \mathrm{F}_s, \mathrm{L}$	Noise	Trigger-less, Backdoor
This paper	$\mathrm{FL}_s, \mathrm{F}_s, \mathrm{L}$	Bagging+Noise	Trigger-less, Backdoor

Plug Eq 32 into Eq 24, for all $0 \le s \le d$, we have

$$
\begin{aligned}
T_{d+1} =& \sum_{u=0}^{d+1}\sum_{v=0}^{d+1} |L(u,v;s,d)|\gamma^u \rho^{d+1-u}(v-u) + \\
& \sum_{u=0}^{d+1}\sum_{v=0}^{d+1} K|L(u-1,v-1;s,d)|\gamma^u \rho^{d+1-u}(v-u) \\
=& \rho \sum_{u=0}^{d}\sum_{v=0}^{d} |L(u,v;s,d)|\gamma^u \rho^{d-u}(v-u) + \qquad (33) \\
& \gamma K \sum_{u=0}^{d}\sum_{v=0}^{d} |L(u,v;s,d)|\gamma^u \rho^{d-u}(v-u) \qquad (34) \\
=& \rho s(\rho-\gamma) + \gamma K s(\rho-\gamma) \\
=& s(\rho-\gamma)
\end{aligned}
$$

We have Eq 33 and Eq 34 because $L(u,v;s,d) = 0$ when $u > d$, $v > d$, $u < 0$, or $v < 0$.

□

E Experiments

E.1 Dataset Details

MNIST is an image classification dataset containing 60,000 training and 10,000 test examples. Each example can be viewed as a vector containing 784 (28×28) features.

CIFAR10 is an image classification dataset containing 50,000 training and 10,000 test examples. Each example can be viewed as a vector containing 3072 ($32 \times 32 \times 3$) features.

EMBER is a malware detection dataset containing 600,000 training and 200,000 test examples. Each example is a vector containing 2,351 features.

Contagio is a malware detection dataset, where each example is a vector containing 135 features. We partition the dataset into 6,000 training and 4,000 test examples.

MNIST-17 is a sub-dataset of MNIST, which contains 13,007 training and 2,163 test examples. MNIST-01 is a sub-dataset of MNIST, which contains 12,665 training and 2,115 test examples. CIFAR10-02 is a sub-dataset of CIFAR10, which contains 10,000 training and 2,000 test examples.

For CIFAR10, we categorize each feature into 4 categories. For the rest of the datasets, we binarize each feature. A special case is L, where we do not categorize features.

E.2 Experiment Results Details

We train all models on a server running Ubuntu 18.04.5 LTS with two V100 32GB GPUs and Intel Xeon Gold 5115 CPUs running at 2.40GHz. For computing the certified radius, we run experiments across hundreds of machines in high throughput computing center.

E.2.1 Defend Trigger-less Attacks

Comparison to Bagging We show full results of comparison on $\texttt{F}_s$ in Figures 3 4 and 5. The results are similar as described in Section 7.

We additionally compare BagFlip with Bagging on $\texttt{FL}_1$ using MNIST-17 and $\texttt{L}$ using MNIST, and show the results in Figure 6. BagFlip still outperforms Bagging on $\texttt{FL}_1$ using MNIST-17. However, Bagging outperforms BagFlip on $\texttt{L}$ because when the attacker is only able to perturb the label, then $s = 1$ is equal to $s = \infty$ and flipping the labels hurts the accuracy.

Comparison to LabelFlip We compare two configurations of BagFlip to LabelFlip using MNIST and show results of BagFlip in Figure 7. The results show that LabelFlip achieves less than 60% normal accuracy, while BagFlip-0.95 (BagFlip-0.9) achieves 89.2% (88.6%) normal accuracy, respectively. BagFlip achieves higher certified accuracy than LabelFlip across all R. In particular, the certified accuracy of LabelFlip drops to less than 20% when $R = 0.83$, while BagFlip-0.95 (BagFlip-0.9) still achieves 38.9% (36.2%) certified accuracy, respectively. **BagFlip has higher normal accuracy and certified accuracy than LabelFlip.**

E.2.2 Defend Backdoor Attacks

We set $k = 100$ for MNIST-17 when comparing to FeatFlip. We set $k = 50, 200$ for MNIST-01 and CIFAR10-02 respectively when comparing to RAB. And we set $k = 100, 1000, 3000, 30$ for MNIST, CIFAR10, EMBER, and Contagio respectively when evaluating BagFlip on $\texttt{F}_1$. We use BadNets to modify 10% of examples in the training set.

Comparison to RAB We show the comparison with full configurations of RAB-σ in Figures 8(a) and 8(b), where $\sigma = 0.5, 1, 2$ are different Gaussian noise levels. Note that RAB's curves are not visible because the certified radius is almost zero anywhere.

Results on EMBER and CIFAR10 BagFlip cannot compute effective certificates for CIFAR10, i.e., the certified accuracy is zero even at $R = 0$, thus we do not show the figure for CIFAR10. Figure 9 shows the results of BagFlip on EMBER. BagFlip cannot compute effective certificates for EMBER, neither. We leave the improvement of BagFlip as a future work.

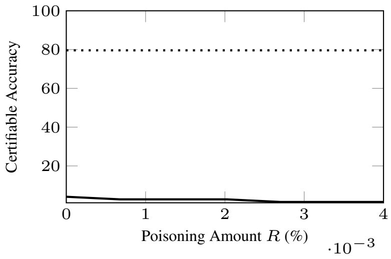
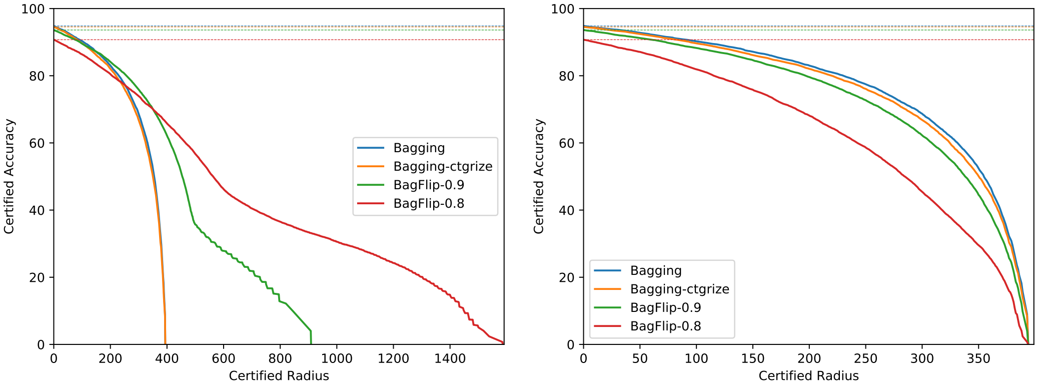
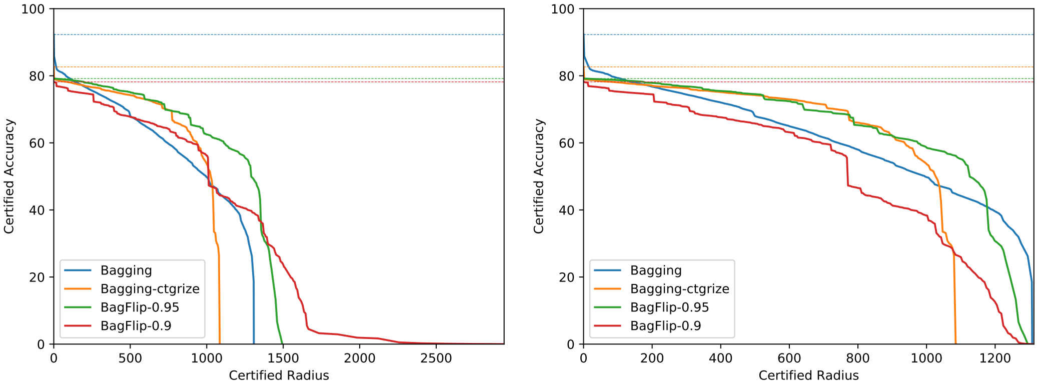


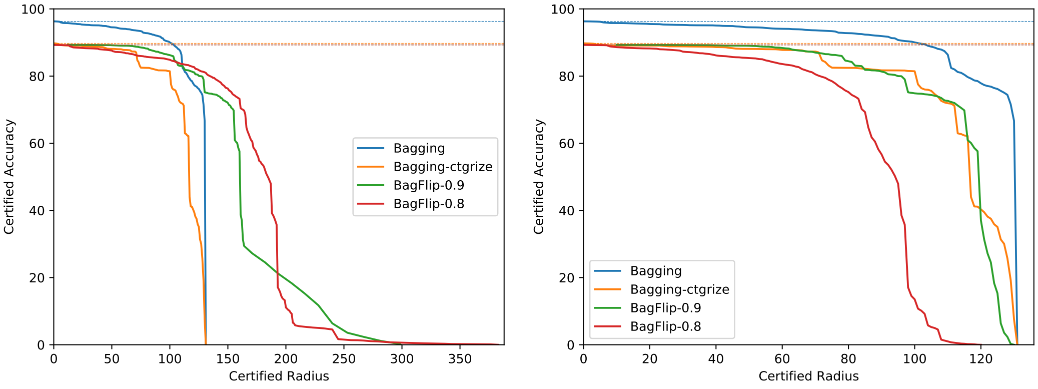
Figure 9: BagFlip-0.95 on EMBER against backdoor attack with $\texttt{F}_1$. Dashed lines show normal accuracy.



(a) F_1 on MNIST (b) F_∞ on MNIST

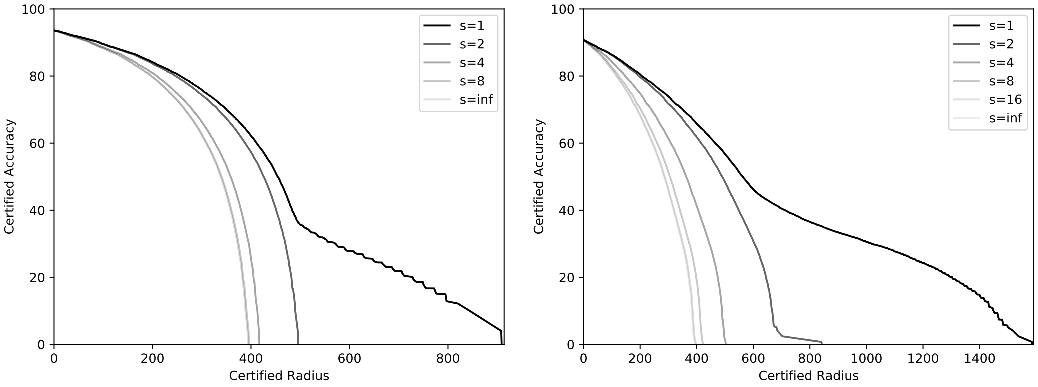


(c) F_1 on EMBER (d) F_∞ on EMBER



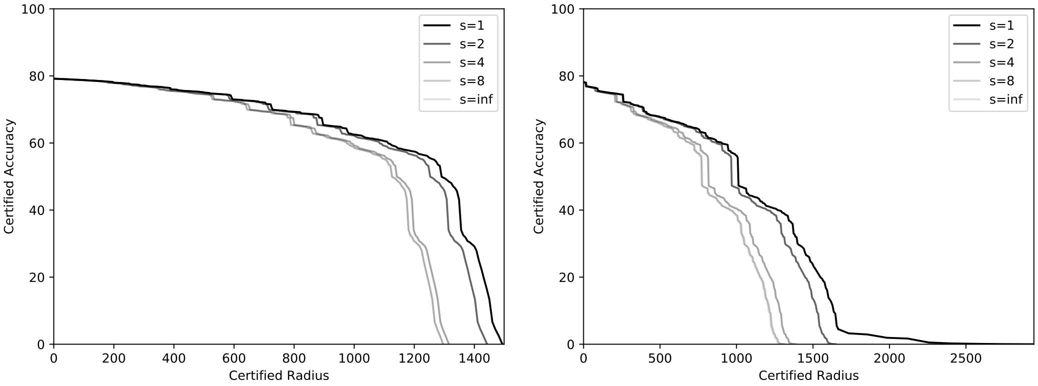
(e) F_1 on Contagio (f) F_∞ on Contagio

Figure 3: Compared to Bagging [15]. The horizontal dashed lines show the normal accuracy. The solid lines show the certified accuracy at different R. BagFlip-a shows the result of the noise level a. The blue line shows the uncategorized version of bagging.



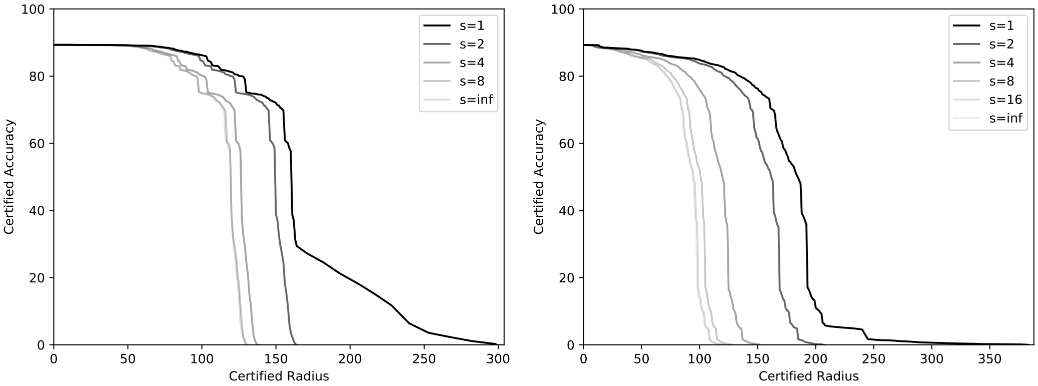
(a) BagFlip-0.9 on MNIST

(b) BagFlip-0.8 on MNIST



(c) BagFlip-0.95 on EMBER

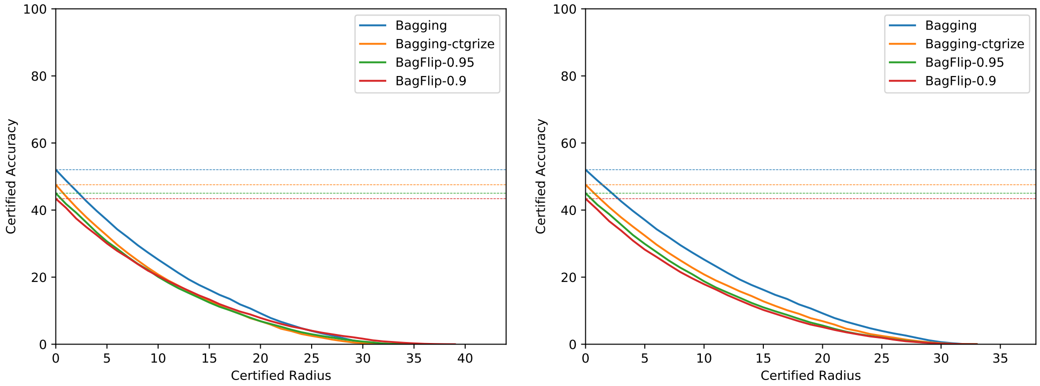
(d) BagFlip-0.9 on EMBER



(e) BagFlip-0.9 on Contagio

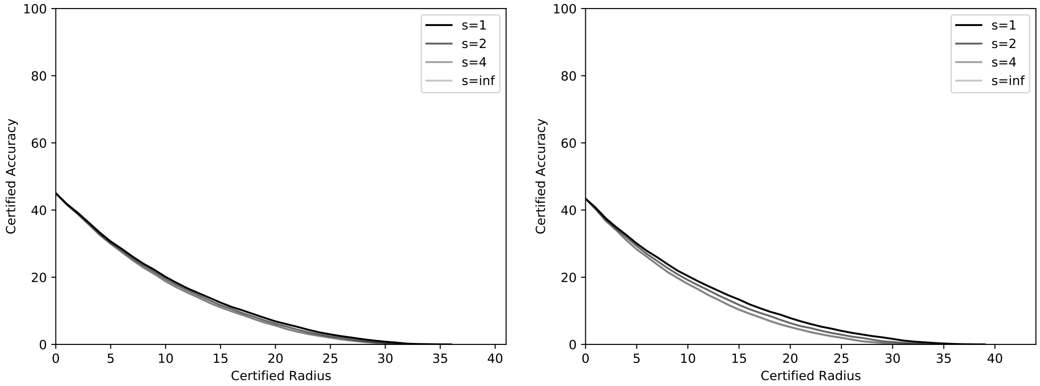
(f) BagFlip-0.8 on Contagio

Figure 4: Results of BagFlip on different s and different datasets.



(a) F_1 on CIFAR10

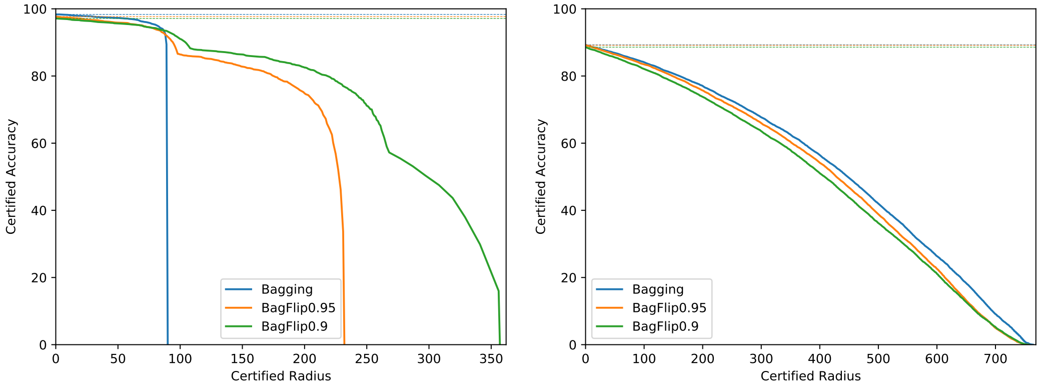
(b) F_1 on CIFAR10



(c) BagFlip-0.95 on CIFAR10

(d) BagFlip-0.9 on CIFAR10

Figure 5: Results of BagFlip on CIFAR10.



(a) Compared to Bagging on FL_1 on MNIST-17

(b) Compared to Bagging on L using MNIST

Figure 6: Compared to Bagging on FL_1 on MNIST-17 and L using MNIST.

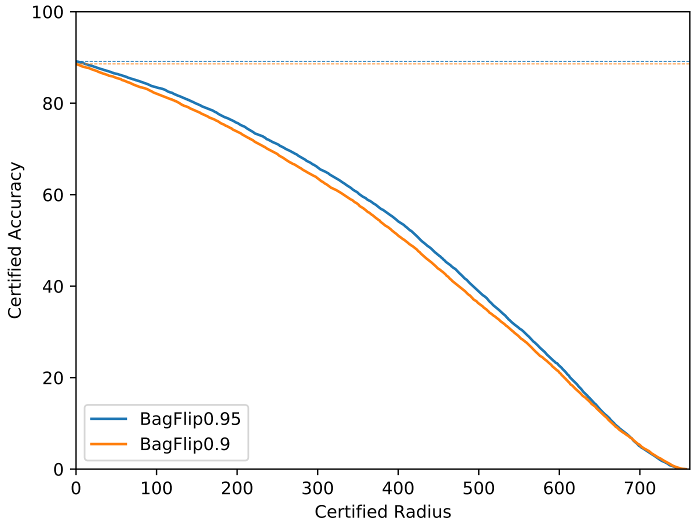
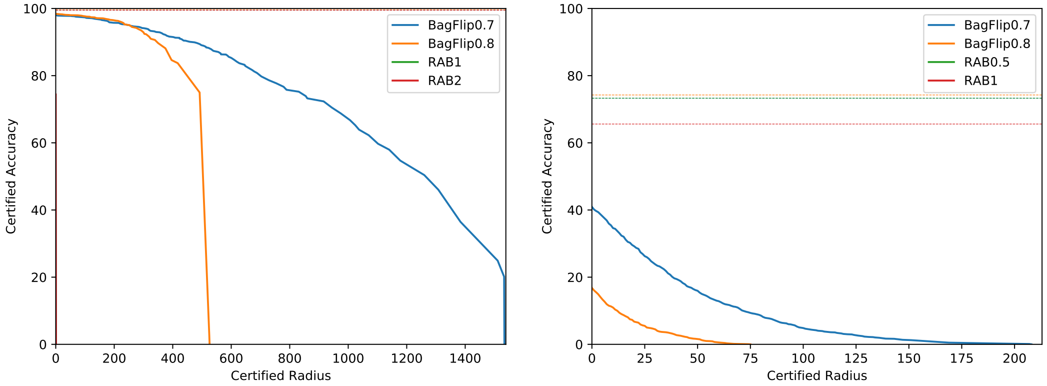
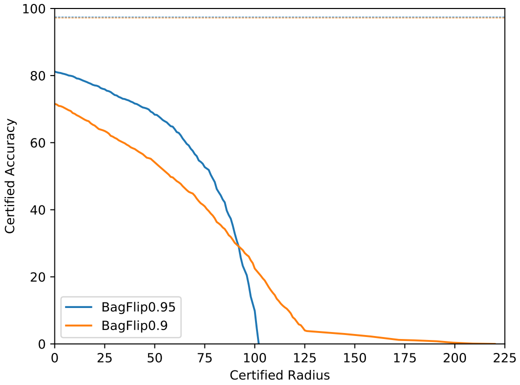


Figure 7: Results of BagFlip on L.



(a) Compared to RAB on MNIST-01

(b) Compared to RAB on CIFAR10-02



(c) Compared to FeatFlip on MNIST-17

Figure 8: Compared to FeatFlip and RAB.