

1 We would like to thank all reviewers for reading our paper and providing constructive comments. Upon acceptance, we  
2 will proofread and fix all editorial issues. Below we first address a few common concerns.

3 **Learning mode and intervention mode.** In most cases, the game designer is expected to first learn about the agents  
4 (e.g. payoff functions) and then use the information (shared weights in the equilibrium layer) to design interventions.  
5 Sometimes, the primary interest is to understand agent behaviors, and hence only the learning mode is needed.  
6 Alternatively, when all game inputs are known, the focus is on the intervention mode. Thus, the two modes are both  
7 independent and integrated from the application point of view. In the final version, we will (i) explain in § 2.1 how these  
8 two modes work in different contexts; and (ii) emphasize this point in the mathematical formulation presented in § 2.2.

9 **Relation between VI and equilibrium.** We agree that it is neither rigorous nor necessary to assert that “most”  
10 equilibrium problems can be formulated as VI. Suffice it to say that VI is a powerful tool to study various equilibrium  
11 problems. In the final version we will (i) stress the connection between monotone VI problems and equilibrium problems  
12 with recommended references; and (ii) provide more examples of the equilibrium problems that can be tackled by VI.

13 **Apply ML in real-world domains.** Our work is inspired by the current interests on complex optimization-based layers.  
14 Although the field has laid a solid theoretical foundation, it has yet to fully exploit the connection between this powerful  
15 architecture and real-world problems. Our work not only adds a novel theoretical concept into the rapidly growing  
16 toolbox, but also demonstrates the potential of applying the end-to-end framework with VI layers in games.

17 **Reviewer 1.** Thanks for your generally positive view of our approach. We would like to emphasize the technical  
18 contribution of our work as follows. It is the first to treat VIs as individual layers in the end-to-end framework. We  
19 propose (i) a new algorithm (tested in Appendix B.3) for solving VI problems in forward propagation and (ii) a new  
20 sensitivity analysis method for VI, which views VI problem as a network of infinite number of QP layers, for backward  
21 propagation. Both methods are novel for VI problems.

22 **Reviewer 2.** Thanks for your comments and suggestions. Upon acceptance, we will (i) carefully separate well known  
23 results included in the analysis from our own results, (ii) treat the second example as a continuing example, and (iii)  
24 clarify the conditions used in (C.3). Below are our responses to the weaknesses identified in your report.

25 **First**, the main focus of our work is to establish the theoretical foundation of differentiable VI layers and explore how  
26 such an end-to-end framework can be used for learning and intervention in games. Thus, the experiments focus on  
27 small examples to validate the methodologies. As equilibrium layers are much more complicated than traditional layers,  
28 it cannot be solved by generic non-linear solvers in MATLAB. **Second**, we respectively disagree with the opinion that  
29 the contribution of our work is weakened by the lack of new mathematical techniques. In fact, we propose, analyze and  
30 validate two new algorithms, one for solving VI problems and the other for sensitivity analysis, as an integral part of the  
31 new framework. We also respectively disagree that Wardrop’s equilibrium brings about too much required structure,  
32 because it is just a special case of the VI based layers. The proposed framework is not restricted in any way by the  
33 structure that comes with that equilibrium problem. **Third**, as explained above, the learning and intervention modes are  
34 independent in the sense of objective functions and only integrated from the application point of view. Both modes are  
35 end-to-end, because the equilibria of games are integrated into the framework as individual layers. The learning mode  
36 itself can be viewed as a generalization of the end-to-end learning framework proposed in reference [33].

37 **Reviewer 3.** Thanks for your constructive comments. Some of your concerns are addressed in the general response.  
38 Below is the answer to an important question raised in your comments.

39 **Central designer.** We agree that in some cases a game may not have a “natural” designer like the auctioneer. We define  
40 the central designer as an authority whose action can influence the outcome of the game. In congestion game, for  
41 instance, the owner of the road network (typically the “government”) has the power to levy toll on roads, or implement  
42 control measures. These actions can be expected to affect how agents behave and eventually the evolution of the game.  
43 We believe that in most (if not all) games, an external designer could be created, if only virtually, to play such a role.

44 **Reviewer 4.** Thanks for your positive view of our work. We will clarify the description about convergence in § 3.2.1.  
45 The paper about AI Economist is actually reference [64], and we will add more details in the literature review.

46 **The explicit differentiation method and the cited method.** Both methods employ a specific solution in the feasible  
47 set (typically a polyhedron) to deal with the non-uniqueness issue. In the cited method, it has to be a nondegenerate  
48 extreme point. However, if such a solution is not available from the equilibrium algorithm, it has to be obtained  
49 separately, which is a burden in the end-to-end optimization. Since our method unrolls the projection method, it only  
50 works when  $F_\lambda$  is monotone and the projection method converges. We will clarify this point in the final version. Thanks  
51 for your advice on how to expand our results to the general monotone setting.

52 **Example 2 and the experiment.** The formulation of learning mode in the experiment is given in Example 2. In  
53 this case,  $f$  represents the number of agents on each edge (flow), which is the only observable state. Typically, it is  
54 difficult to directly measure/observe the underlying parameters, such as the time value of money and the coefficients in  
55 the hidden costs functions. Instead, they are estimated by minimizing the loss, i.e., the sum of squared errors of the  
56 equilibrium flows. We report the losses on log scale for better visualization in Figure 7.