## (Probably) Concave Graph Matching Supplementary material

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## 1 Frank-Wolfe with concave search

An orthogonal basis to  $lin(\mathcal{F})$  is computed similarly to Lemma 1 in the paper:

**Lemma 1** (orthonormal basis for one-sided permutations). If the columns of  $F \in \mathbb{R}^{n_0 \times (n_0-1)}$  form an orthonormal basis for  $\mathbf{1}^{\perp}$  in  $\mathbb{R}^{n_0}$  then the columns of  $F \otimes I_n$  are an orthonormal basis for  $\ln(\mathcal{F})$ .

The energy  $E_2(X)$  in this case does not model the matching problem well since it gives rise to trivial solutions. Instead, we chose to optimize a similar energy (Solomon et al., 2016):  $E(X) = \sum_{ijkl} X_{ij} X_{kl} (A_{ik} - B_{jl})^2$ . This energy can also be written in matrix form:  $[X]^T M[X]$  where  $M = -2B \otimes A + 11^T \otimes A^2 + B^2 \otimes 11^T$  (where  $C^2$  implies entry-wise operation) and after restricting it to  $\lim(\mathcal{F})$  its Hessian is of the form  $-2FBF \otimes A + FB^2F \otimes 11^T$ . Assuming A, B are Euclidean distance matrices, the right summand is negative semidefinite, but the left summand is not. This is because that A is not conjugated by F: it has a large positive eigenvalue as a result of the Perron-Frobenius Theorem.

The linear program solved in each iteration of the algorithm takes a surprisingly simple form: it amounts to solving  $\min_{X \in \text{hull}(\mathcal{F})} \text{tr}(\nabla E(X_0)^T X)$  which can be solved simply by assigning the value 1 to the index of the minimal value in each row of  $\nabla E(X_0)$ . This procedure always outputs solutions in  $\mathcal{F}$ .

The convex energies we subtract from the objective during the concave search should be constant on  $\mathcal{F}$  so a reduction in the subtracted energy is the same as in the original energy E(X). We use the quadratic form defined by  $\lambda * \Lambda$  where  $\Lambda$  is a  $nn_0 \times nn_0$  diagonal matrix defined by  $D_{ijij} = \max_j \{\sum_{kl} |M_{ijkl}|\}$ . D is a positive definite matrix and for  $\lambda = 1$ , W - D is guaranteed to be negative semidefinite. The values of  $\lambda$  need not be discretized since there are only n different critical values - the ones that change the minimum calculation mentioned in the previous paragraph.

## References

Solomon, J., Peyré, G., Kim, V. G., and Sra, S. (2016). Entropic metric alignment for correspondence problems. ACM Transactions on Graphics (TOG), 35(4):72.