

## A Proof of Lemma 3.1 and Theorem 3.1

**Proof A.1** Consider the model given in the following equations:

$$\begin{aligned}\mathbf{x}_t &= \mathbf{y}_t + \boldsymbol{\eta}_t \\ \mathbf{y}_t &= \mathbf{A}_0 \mathbf{y}_t + \sum_{\tau=1}^L \mathbf{B}_\tau \mathbf{y}_{t-\tau} + \boldsymbol{\epsilon}_t\end{aligned}\tag{A1}$$

Let  $\mathbf{A} = (\mathbf{I} - \mathbf{A}_0)^{-1}$  and  $\mathbf{v}_t = \mathbf{A}^{-1} \mathbf{y}_t$ . By simple re-parameterization, we have

$$\begin{aligned}\mathbf{x}_t &= \mathbf{A} \mathbf{v}_t + \boldsymbol{\eta}_t \\ \mathbf{v}_t &= \sum_{\tau=1}^L \mathbf{B}_\tau \mathbf{A} \mathbf{v}_{t-\tau} + \boldsymbol{\epsilon}_t\end{aligned}\tag{A2}$$

Actually, the model after re-parameterization given in Equations A2 is the state-space model investigated by Zhang and Hyvärinen in [24]. Thus, we can follow the idea presented in [24] to prove the identifiability of our model.