

1 We thank all reviewers for their time and constructive comments.

2 **Common Concerns.** We first address concerns that were brought up by multiple reviewers.

- 3 • **(R 1, 2) Tractability and Runtime.** Since we use the CNF/ Neural ODE solver, our runtime is comparable to  
4 that of CNF, and thus we can use advances such as FFJORD [18] to scale with manifold dimension. Also note  
5 the log-determinant formulas for tested manifolds scale linearly in dimension, giving us tractability. PRNVP  
6 is faster as it uses the (non-continuous) RealNVP in the projected space. Also, our experiments show that  
7 NMODE is more sample efficient than other methods (Appendix C.2, first paragraph), so for density estimation  
8 we can use substantially fewer batches than other methods; this results in lower overall runtime.
- 9 • **(R 1, 2) Choice of  $\epsilon$ .** Appendix D.1 discusses controlling the dynamics with  $\epsilon$ . With a bound on the derivative  
10 we enforce a Lipschitz constraint on the NMODE solutions, in which case  $\epsilon$  can be chosen less than  $r/L$ ,  
11 where  $L$  is the Lipschitz constant and  $r$  is the local radius of injectivity.  $\epsilon$  can be arbitrary for hyperbolic  
12 space (since the radius of injectivity is infinite) and should be less than  $\pi/L$  for the sphere. We find that most  
13 reasonably small choices work in practice. The quantifier for Prop 5.1 should be "for some"; this will be fixed.
- 14 • **(R 2, 3, 4) Applications and Concerns about Results** First note that aside from the general use cases  
15 presented in the paper (to data that does not require a manifold constraint), our approach gives a way to learn  
16 manifold-constrained flows for problems that require this constraint. In physics, we allow learning tractable  
17 densities on unitary Lie groups for Lattice Quantum Field Theory [37], in biology we need these densities  
18 for protein-structure prediction [21], and in robotics we need these for path navigation/motion estimation  
19 (e.g. modeling motion of a robot arm as a density in  $\mathbb{T}^6$ ) [13]. Regarding our results, the lower margins with  
20 respect to Euclidean baselines as we increase dimension are due to the fact that it is possible to compensate for  
21 geometric distortion in higher Euclidean dimensions (Nickel et al., 2017). Note that for small dimensions (e.g.  
22 2) our results significantly exceed the baselines and this is important for application via faster inference (more  
23 efficient modeling of data in low dimensions). We also note that although the example densities are simplified  
24 cases, we outperform all state-of-the-art baselines, highlighting existing flaws that we resolve.
- 25 • **(R 2, 4) Choice and Construction of Local Charts** The manifold is determined a priori in many applications,  
26 as in all applications above. Thus the local charts are determined a priori when the requisite manifold is  
27 selected and the construction follows from the manifold definition. Regarding "...choice of the charts for  
28 non-Riemannian manifolds...", note that the manifolds considered in our paper are smooth, hence admit a  
29 Riemannian metric, and are thus Riemannian. Therefore all manifolds we consider have well-defined charts.

30 **Reviewer 1** Thank you for your kind words on our approach and the potential future impact of our paper.

- 31 • **"...the log likelihood...worse than those reported in...[2]."** We used [2]'s code (we had access before they  
32 open sourced it), fixed bugs in their implementation (that the authors confirmed to be actual bugs), and also fix  
33 evaluation (we run with a validation set, unlike [2], which gives misleadingly high numbers).
- 34 • **"Regarding the claims of generality..."** and **"I was confused by some parts of the motivations."** Yes, both  
35 (exp map and chart) are equally general. Projecting from the ambient space is not as principled (projecting  $\mathbb{R}^3$   
36 to  $\mathbb{S}^2$  is not well-defined) whereas our exponential map helps learn over the space directly. We claim to be the  
37 first general principled method; we will augment the paper to make this more clear.

38 **Reviewer 2** We are glad you appreciated the theoretical aspects and motivation of our paper.

- 39 • **"Does the method really benefit from using more complex solvers and the adjoint method in this case?"**  
40 Note that other methods do not provide the ability to incorporate more complex solvers from the classical  
41 literature, yet we do. Certainly, switching to one of these solvers will not always help (as in the case you  
42 mentioned). However,  $\epsilon$  can frequently be fairly large; in these cases, a more complex solver would help.

43 **Reviewer 3** Thank you for recognizing our theoretical and algorithmic merits.

- 44 • **"determinant of the derivative of the exponential map is needed...this may be difficult beyond the most  
45 simple manifold..."** This is possible given a parameterization of the manifold via local charts.
- 46 • **"...exponential maps are used...this should be stated much more explicitly..."** This will be done.
- 47 • **"Thm 4.1 follows from [4] due to the ambient assumption?"** Yes, the proof is analogous to that of [4].
- 48 • **"...the Whitney embedding Theorem is not isometric..."** NMODE itself is not affected by the change of  
49 metric. For MCNF, note that for relevant manifolds (e.g. those for applications given above: hyperboloid,  
50 sphere, unitary Lie groups, etc.) the data has an explicit embedding.

51 **Reviewer 4** Thank you for your comments and appreciation of our usage of charts.

- 52 • **"...whether VAE can learn the local chart structure and the Riemannian metric..."** The VAE does not  
53 learn the manifold structure; the hyperbolic manifold is chosen a priori.