

1 We greatly appreciate the reviewers' effort and helpful comments. We will fix the typos and polish the writing by  
2 incorporating the reviewers' suggestions.

### 3 **Response to Reviewer #1**

4 **Comment 1:** "The significance of the proposed method is not very clear..."

5 **Response 1:** First, the question of solving saddle-point problems using only projection-free methods is interesting  
6 (Reviewer #3 also mentions this point). It also has great theoretical significance in the optimization area.

7 Secondly, though our analysis is specified for the convex-strongly-concave setting, there is a simple way to adopt  
8 our algorithms to solve the general convex-concave saddle point problems. For a convex-concave function  $f(\mathbf{x}, \mathbf{y})$ ,  
9 we can construct a convex-strongly-concave function as  $f_\epsilon(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}, \mathbf{y}) - \epsilon\|\mathbf{y} - \mathbf{y}_0\|^2$  and solve  $f_\epsilon(\mathbf{x}, \mathbf{y})$  by our  
10 algorithms (such as MPCGS). Though the convergence rate of this method could be suboptimal, it's a practical way to  
11 deal with the general convex-concave situations.

12 In addition, [6] shows some examples of saddle point algorithms where projection onto the constrain sets is hard. These  
13 applications includes robust optimization, two-player games and sparse structured SVM.

14 **Comment 2:** "Why do we consider nuclear norm constraint for this classification problem?"

15 **Response 2:** Nuclear norm is a popular penalty in multi-class classification because datasets with many categories  
16 usually exhibit low-rank embedding of the classes behaviour (see [4]).

17 **Comment 3:** "(arXiv:1804.08554, section 5.4 and 5.6) can be added."

18 **Response 3:** We find that this paper does not have section 5.4 and 5.6. Also, it is irrelevant to our paper. Perhaps you  
19 give the wrong paper id.

20 **Comment 4:** "The presentation is mostly clear, but some parts are lacking important details."

21 **Response 4:** We will modify the confused sentences and clarify our results.

22 **Comment 5:** "Line 116: the linear optimization on  $Xc$  only needs to find the top singular vector of  $X$ , which only costs  
23  $O(\text{nnz}(X))$  time. This statement is inaccurate."

24 **Response 5:** You're right. The complexity should be  $\tilde{O}(\frac{N}{\sqrt{\epsilon}})$ , where  $N$  is the number of non-zero entries in the gradient.

### 25 **Response to Reviewer #3**

26 **Comment 1:** "It is not clear why the assumption that the objective is strongly concave is needed."

27 **Response 1:** Notice that we adopt CGS algorithm to approximately solve a concave problem in Alg 4 (line 3). When  
28 the objective is strongly concave, the CGS method only requires to call  $\sqrt{\kappa} \log(1/\epsilon)$  SFO. When the objective is not  
29 strongly concave, the CGS method requires to call  $1/\sqrt{\epsilon}$  SFO. The convergence rate of CGS will significantly influence  
30 the total number of iterations of our algorithm because CGS is performed in the inner loop.

31 **Comment 2:** "It seems that the bounds are loose at several points."

32 **Response 2:** For our algorithms, we think our bounds are almost tight. We think that there exists better algorithms  
33 which only requires to call  $O(1/\epsilon)$  LO as the projection-free algorithms for the convex optimization, but finding such  
34 an algorithm is a big challenge because minimax problem is much more complicate than the minimization problem.

35 **Comment 3:** "Line 4 of Alg 3: not clear what we get  $v_k$  here as one of the outputs of prox-step if its updated in the  
36 following line via CndG"

37 **Response 3:** Actually, we do not compute  $v_k$  via CndG. We only update  $x_k, y_k$  and  $v_k$  by the prox-step. According to  
38 Alg 3 (the procedure of prox-step), the results of the prox-step guarantee that  $x_k, y_k$  and  $v_k$  satisfies the equations and  
39 inequality in the Line 4 of Alg 3.

### 40 **Response to Reviewer #6**

41 **Comment 1:** "L40 is a bit of an over-claim".

42 **Response 1:** We will modify the over-claim sentences and clarify our setting. On the other hand, there is a simple way  
43 to adapt our methods to the convex-concave setting (see the second paragraph of the Response 1 to Reviewer #1).

44 **Comment 2:** "I am a bit confused about Remark 2. since when  $\epsilon$  is small we could have  $\log(1/\epsilon) \gg \sqrt{\kappa}$ . Moreover,  
45 isn't the condition you would like to require  $\sqrt{\kappa}/\epsilon \gg \kappa^2$ ?"

46 **Response 2:** The condition should be  $2^{-\sqrt{\kappa}} < \epsilon < \kappa^{-1.5}$ . Then we can get  $(\sqrt{\kappa}/\epsilon + \kappa^2) \log(1/\epsilon) < \kappa/\epsilon$ .

47 **Comment 3:** "SVRE has no-guarantees in the convex-strongly-concave setting."

48 **Response 3:** To our knowledge, there is no stochastic projection algorithm has guarantees in the convex-strongly-  
49 concave setting. On the other hand, we have already took a nuclear norm regularization. Usually it does not need  
50 additional L2 regularization.