General We would like to thank all three reviewers for their suggestions – we've made these updates in our internal version and we believe the updated version is stronger as a result. We note reviewers were positive regarding novelty and significance, noting that the "theoretical and empirical analysis are new" and that "showing that unlabeled data alleviate[s] this problem [of robust sample complexity] is crucial because it is much easier (and cheaper) to collect."
Reviewers raised several questions regarding extensions of theoretical and empirical results, the relationship between

6 theory and practice, and implementation clarity. We have investigated these, and briefly summarize our updates below.



- **R1** Different threat models. For  $L_2$  robustness, we observe similar improvements from UAT, which we've added to the appendix. For example, on CIFAR-10 at  $L_2$  radius  $\epsilon = 0.87$ , with 4K labeled and 32K unlabeled examples, the purely supervised model achieves 32.7% robust accuracy, the supervised oracle achieves 53.9%, and UAT almost matches this, with 55.2%. This represents a 21% absolute gain from using unlabeled data, which captures over 90% of the oracle
- improvement, without using labels. We observe similar results for  $\epsilon = 0.435$ , of 47.3% / 70.3% / 66.3% respectively.
- 12 Theoretical analysis of UAT-OT. It's a good question. We have focused in the paper on UAT-FT, since it performs
- <sup>13</sup> significantly better in our experiments. We are confident a similar result should hold for UAT-OT, using a single label. <sup>14</sup> The rough intuition is that, with many unlabeled examples, the OT loss ensures that  $|\langle \hat{w}, \theta^* \rangle| / ||w||$  is large, and the
- Fine rough interview is unit, with many undecred examples, the OT ross ensures that  $|\langle w, v \rangle / / ||w||$  is targe, and c is single label is necessary only to determine the sign of  $\langle \hat{w}, \theta^* \rangle$ . We will add a comment on the analysis of UAT-OT.
- **R2** What is the proper amount of unlabeled data m? We understand this as two questions how theory explains our
- 16 **R2** What is the proper amount of unlabeled data m? We understand this as two questions how theory explains our 17 empirical observations, and why VAT outperforms UAT++, on SVHN when m is small. For the first question, the
- theory suggests performance should increase monotonically with m indeed, our experiments validate that unlabeled
- data always helps, and that larger m strictly improves performance. Regarding the second, we find UAT++ outperforms
- 20 VAT when hyperparameters are properly tuned. In our original SVHN experiments, we directly reused CIFAR-10
- hyperparameters. We find it assuring that even with zero hyperparameter tuning, these original results are qualitatively
- 22 consistent: using unlabeled data provides vast improvements over labeled data alone, and UAT++ outperforms baselines,
- particularly in the  $m \gg n$  case we view as important for practical applications. After re-tuning learning rates for all
- <sup>24</sup> models, the figure above shows that UAT++ outperforms VAT for all m.
- 25 Performance drop with increasing m in Table 1. You're right that UAT is not robust to arbitrarily out-of-distribution
- <sup>26</sup> unlabeled examples, and we've clarified the text accordingly. Qualitatively, we observe greater distribution shift in
- 80m@500K than in 80m@200K. Our main point is that UAT is moderately robust to distribution shift, sufficient for
- us to leverage uncurated data to achieve SOTA robust accuracy. This is indeed only a first step we're excited to see
- <sup>29</sup> <u>future research into more effective ways to leverage uncurated, and further out-of-distribution data.</u>
- R3 *Related literature*. We agree that the three papers mentioned provide useful perspectives on robust sample complexity.
   Thanks for mentioning them we've expanded the discussion of this in the updated paper.
- Pseudocode. We agree we've significantly updated our appendix with these details, notably pseudocode, but also experimental procedures, hyperparameters, ablations, and negative results. We've added a significant section on
- experimental procedures, hyperparameters, ablations, and negative results. We've added a significant section on pseudocode, which we can't include here for space. Algorithm 1 shows the UAT++ update (the others are similar). To
- simplify notation, when writing  $(x, y) \sim U_m$ , the target y is always the fixed target pseudo-label.  $\hat{\mathcal{L}}^{adv}$  and  $\hat{\mathcal{L}}^{OT}$  are the
- empirical estimates of the robust loss from Madry et al, 2017 (as in UAT-FT) and  $\mathcal{L}^{OT}$  respectively. These loss terms are
- also further detailed in our updated appendix.
- How many labels are required? Short answer: not many. Most papers train with 50K labels on CIFAR-10, but we show
- that using UAT allows going from 36K to 4K labels, while maintaining adversarial accuracy robust accuracy against 55.5% to 54.1% J
- <sup>40</sup> FGSM-20 only decreases from 55.5% to 54.1%. Long answer: in the Gaussian model, provided sufficient unlabeled
- 41 data, only a single label is necessary. Qualitatively, the theoretical model suggests that UAT only needs sufficient
- labels for natural generalization (as opposed to robust generalization), which in the Gaussian model is just a single
   label. To study this in practice, we tried pushing to even lower label regimes on CIFAR-10, and still observe significant
- label. To study this in practice, we tried pushing to even lower label regimes on CIFAR-10, and still observe significan
   adversarial accuracy (now added to appendix): 2K labels yields 51.9%, and 1K yields 47.7% under FGSM-20.