

1 **Reviewer #1** We thank the reviewer for recognizing the contribution of our paper, and providing valuable comments.

2 1) Yes, there exist some value-based methods for general zero-sum Markov games (see references [29-36] in the  
3 paper). However, except [36], these works only considered *finite action* spaces (or even finite state spaces), which  
4 do not apply to the LQ game with *continuous* state-action spaces. Moreover, some of the algorithms, e.g., those in  
5 [32,33,34], are batch RL algorithms. In contrast, our PO algorithms can handle continuous action spaces, and can be  
6 implemented online. These are in fact some of the benefits of PO methods compared with value-based ones. We will  
7 emphasize these points in the final version. We also agree that it is beneficial to compare empirically with the algorithm  
8 in [36], the only existing value-based online method for our setting. We will add this comparison in the final version.

9 2) The projection is mainly to guarantee  $Q - L^\top R^\nu L > 0$  along the iterations. This is necessary for the convergence  
10 of the inner-loop updates, and the stability of the control pair  $(K(L), L)$ . In fact, we have indeed observed in some  
11 simulation examples, though very rare, that although at the NE Assumption 2.1 ii) holds, the outer-loop iterate  $L$   
12 still converges to  $L^*$  without  $Q - L^\top R^\nu L > 0$  along the iterations. However, the inner-loop update for the inner  
13 LQR problem may not converge in that case. We will make the empirical results more exhaustive, by adding these  
14 observations, together with more careful discussions on the projection in the final version.

15 **Reviewer #2** We thank the reviewer for finding our paper well-written and clear. However, we would like to argue that  
16 the comments are totally unfair and inaccurate, as can easily be seen by comparing the two submissions very carefully.

17 1) The two papers are indeed quite different. After viewing these comments, the 1st author checked out the draft of  
18 5637, which indeed shares one common author with this paper. After careful reading, we believe that the comments are  
19 unfair since: i) the problems addressed are different: we consider the landscape and PO methods for general zero-sum  
20 LQ games, while 5637 is motivated by solving risk-sensitive RL problems, and LQ game is only one way to reformulate  
21 it for algorithm-design; ii) the algorithms are different: we consider three *model-based* PO methods since we are the  
22 first to consider PO for this setting; while 5637 developed *model-free* actor-critic algorithms, as a follow-up; iii) more  
23 importantly, as theory papers, the techniques in 5637 for proving convergence of *model-free* methods are very different  
24 from the ones here; iv) this paper has been compared and cited properly and anonymously in 5637, see [4] therein.

25 2) The main contributor of the two papers come from two different research groups. When preparing the work, the  
26 main contributor of this paper, i.e., the 1st author, and the 3rd author, were not involved in any way in any part of the  
27 preparation of 5637. There is no overlap of interest for them at all.

28 3) This paper is obviously a more fundamental work. We have a well-motivated and self-contained story, a fundamental  
29 setting, a well-prepared introduction, the 1st study of the landscape, new PO algorithms, solid proofs for convergence,  
30 and empirical results to verify the theory. In contrast, 5637 is a follow-up based on some results here, but has its own  
31 new theoretical contributions, e.g., reformulation of RSRL, convergence of *model-free* methods, etc. This paper has  
32 been completed earlier (than 5637) and posted online right after NeurIPS. The online version has been acknowledged  
33 and cited by other researchers already. We believe that in the long term, this work is beneficial for the whole community.  
34 With these in mind, we would like the reviewer to re-evaluate the novel contribution of this work, instead of viewing  
35 our contributions as something “in addition” to 5637. It is in fact the other way around. To fully address the concern,  
36 we have agreed to remove the only common author on this paper in the final version in order to resolve the conflict of  
37 interest, if that is possible and helps in arriving at the final decision. We sincerely hope our response can change the  
38 reviewer’s viewpoint, and lead to fairer judgments.

39 **Reviewer #4** We thank the reviewer for the very positive comments. Yes, we agree that this work is specific to the  
40 LQ setting. However, first, as an initial step towards developing *PO* methods for Markov games *with convergence*  
41 *guarantees*, the LQ setting seems to be a standard choice, for both theoretical interest and sanity-check, as the case of  
42 LQR for single-agent RL [24]. Second, our LQ setting is indeed the 1st one concerned with the type of Markov games  
43 with *continuous* spaces where RL algorithms (including value-based ones) have convergence guarantees. Third, we  
44 would like to emphasize that zero-sum LQ games per se play important roles in robust control/RL [15,37]. Thus, this  
45 work may provide theoretical foundations for new RL algorithms for robust control synthesis.

46 Under Assumption 2.1 ii), there always exists a set  $\Omega$  that contains the NE, namely, the issue is nonexistent for the LQ  
47 games we consider, which corresponds to an important robust control setting with relatively small disturbances [15].  
48 Without the assumption and the projection, stability of the control pair  $(K(L), L)$  may not hold. We have included the  
49 analysis for this case in our future work.

50 **Reviewer #6** We thank the reviewer for the very positive comments. First, to our knowledge, we are indeed *the first* to  
51 consider the optimization landscape of zero-sum LQ games. Although the convergence rates have all been explicitly  
52 characterized, the order is not explicitly seen from the rates. For example, even for the local linear rates, see Eq. (B.48),  
53 (B.52), and the one below (B.60), the  $(1-\text{rates})$  differ by a factor of  $\mu$  and  $\nu$ , which are the smallest eigenvalues of  $\Sigma_0$   
54 and  $W_{L^*}$ , respectively, and not necessarily smaller/greater than 1. Thus, it is not explicit which one is faster.

55 For general (nonlinear) continuous control, it is fundamentally hard to characterize the robust controller, let alone to  
56 find it via PO. However, it is always possible to *linearize* the general dynamics, so that our results apply at least locally  
57 around the linearization point. Besides, without care for theory, the nested-gradient (double-loop) idea can definitely be  
58 applied for PO for general robust control synthesis. We will add some empirical comparisons on this in the final version.