

	ResNet-4		ResNet-10		ResNet-14	
	Min / Max	Avg	Min / Max	Avg	Min / Max	Avg
Baseline	76.47%/77.35%	76.95%	87.79%/88.52%	88.10%	91.21%/92.12%	91.68%
NeuralODE[8]	52.58%/56.65%	54.47%	68.71%/70.48%	69.43%	75.32%/76.79%	76.13%
ANODE	77.13%/78.00%	77.54%	88.39%/88.87%	88.70%	91.66%/92.13%	91.90%
ANODEV2	77.55%/78.07%	77.83%	88.82%/89.19%	88.97%	92.13%/92.35%	92.19%

Table-R 1: We report results for Neural ODE[8], ANODE[9], and ANODEV2 (using configuration 2) for various models on Cifar-10. We report results where we concatenate time as an additional channel as requested by **R2**. All results were repeated five times with different random seeds. As one can see, ANODEV2 provides statistically consistent improvements. Also, for dynamical problems as shown in Fig-R 1, you can see a clear advantage of ANODEV2 as compared to ANODE[9] (and Neural ODE[8]).

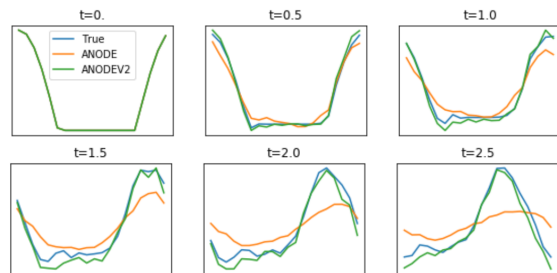


Figure-R 1: Reconstruction of a signal transport problem. Here the task is to predict the change of the input signal (shown in the top left at $t = 0$) in time. The governing equation is a first-order wave equation with variable velocity in time. The blue curve shows ground truth, orange shows ANODE, and green shows ANODEV2. We used a single layer model to learn the transport equation. ANODEV2 performs better, as it can capture the transport physics as a constraint through the Turing’s reaction operator. The x-axis shows spatial location, and y-axis is signal amplitude.

1 We thank all the reviewers for taking the time to review our work and provide their constructive feedback.

- 2 1. **R1/R2:** Elaborate more on the advantage of designing the PDE in this specific way? and show effectiveness on
3 learning dynamical systems, similar to Neural ODE paper. **A:** For vision based problems, Turing’s reaction-diffusion
4 model (with advection) can simulate several different filter shapes, as shown in Figs. 1 and 3 in the paper. This
5 allows us to use the evolution kernel to capture specific features of the problem. For example, we can directly capture
6 a variable velocity wave equation, using the reaction term in the evolution kernel. This is illustrated in Figure-R. 1
7 above, where we test a simple transport phenomena with variable velocity. The governing equation here is $dz/dt =$
8 $c(t) dz/dx$, where $c(t)$ is variable velocity, and z is the signal that changes in time. The learning task is to predict
9 how the signal changes in time. That is we are given $z(t=0)$ and want to infer $z(t)$ at different time points. We test a
10 one layer model in 1D and illustrate the results in Fig.-R 1 above. ANODEV2 can easily capture variable velocity
11 ($c(t)$) with only a single layer through reaction operator, and as you can see the quality of its prediction is better
12 than ANODE. We emphasize that this is a simple problem, and we are now investigating more complex physics
13 based problems, for which we anticipate ANODEV2 to perform better by incorporating physical constraints in the
14 evolution kernel. We will add this result in the paper. We should also note that the evolution kernel does not have to
15 be based on Turing’s system, and other kernels could be used depending on the target application.
- 16 2. **R2:** Unclear whether introducing time-varying weights is better than neural ODE paper which adds time through an
17 extra channel. **A:** With the evolution kernel we can encapsulate different filters without having to store additional
18 filters in memory, which is not possible through time concatenation as performed in [8]. This was illustrated in Figs.
19 1 and 3 in the paper. We have also added an ablation study in Tab-R 1 above which compares ANODEV2 with
20 time concatenation as done in [8]. For fair comparison we have also included results with ANODE which addresses
21 numerical instability of [8]. All the results were repeated five times and we report the statistics. ANODEV2 provides
22 small but statistically consistent improvements (please note that we did not perform any hyper-parameter tuning).
- 23 3. **R2/R3:** Overall the improvements over Neural ODE seem smallish but consistent. **A:** That is true but the improve-
24 ments are statistically consistent. We should mention that there are various factors that need to be studied, for
25 instance initialization of the evolution kernel parameters, which can play an important role in the final generalization
26 of the model. This is a non-convex optimization, and initialization as well as hyper-parameter selections can affect
27 the results. Please note that this is the very first work in this area. see above ANODEV2 performs significantly better
28 for dynamical systems, and this is also without tuning. We anticipate that having a NN model with evolutionary
29 kernel could be very useful for physics based learning problems.
- 30 4. **R2:** (i) The Baseline description (ii) integrators used (iii) Timings report **A:** The baseline is basically a residual
31 network which is equivalent to $N_t = 1$ without parameter evolution. We used Euler time stepping for our experiments
32 but our code in [10] supports (Runge Kutta) RK2, RK4, and RK45 integrators. In terms of flops/timing we have
33 almost the same cost as Neural ODE, but we have the additional cost of evolution operator. However, we emphasize
34 that the latter cost is a lower order term. We will add timings.
- 35 5. **R3:** The central contribution hinges on the mentioned problem of neural ODEs. **A:** We kindly note that this is not the
36 case. Neural ODE provides a potentially weak baseline, and so in addition to comparing to it in Tab.-R 1 above, we
37 also compare to ANODE which is the corrected version of it. As you can see, Neural ODE results are significantly
38 worse than ANODE/ANODEV2. All results in the paper were reported with ANODE to allow for fair comparison.
- 39 6. **R3:** The derivation of optimality for the coupled ODE is interesting but standard from dynamical systems. **A:** That’s
40 a fair point. We applied known methods to derive the optimality conditions. We will clarify this in the paper.
- 41 7. **R3:** Is reference 9 a peer-reviewed paper? **A:** Reference [9] appeared on arxiv at the time of submission of this
42 paper, but is now listed as an accepted paper in IJCAI 2019 conference.