

1 We are grateful for all the reviewers' valuable suggestions and questions. We start by showing some additional  
2 experiments for deep nonlinear ResNets. Consider a nonlinear ResNet  $f(x; \theta) := w^T z_L$  with  $z_L$  recursively defined as

$$z_0 = V_0 x; \quad z_l = z_{l-1} + U_l \sigma(V_l z_{l-1}), \quad l = 1, \dots, L$$

3 where  $V_0 \in \mathbb{R}^{D \times d}$ ,  $U_l \in \mathbb{R}^{D \times m}$ ,  $V_l \in \mathbb{R}^{m \times D}$  and  $w \in \mathbb{R}^D$ . We  
4 test two initializations: (1) standard Xavier initialization; (2) modified  
5 zero-asymmetry(mZAS) initialization :  $U_l = 0$ ,  $w = 0$  and  $(V_l)_{i,j} \sim \mathcal{N}(0, 1/D)$ . The experiments are conducted on Fashion-MNIST, where we  
6 select 1000 training samples forming the new training set to speed up the  
7 computation. The results are displayed in Figure 1.

8 We can see that mZAS initialization always outperforms the Xavier initialization.  
9 Moreover, GD with mZAS initialization is able to successfully  
10 optimize a 10000-layer ResNet. It is clearly demonstrated that ZAS-type  
11 initialization can be helpful for optimizing deep nonlinear ResNets. How-  
12 ever, to make this initialization practical for real scenarios such as ImageNet  
13 still requires more efforts, which is beyond the scope of this paper. We will  
14 add a section in the paper to discuss how to adapt it for nonlinear residual  
15 network and provide some preliminary experiments.

17 **For Reviewer 2:** (1) (for continuous-time GD:  $R(t) \leq \exp(-2\alpha^{2(L-1)}t)R(0)$ ) Thanks to the reviewer for pointing  
18 out this interesting phenomenon that we did not notice it before. It increases the gradient by  $\alpha^{2(L-1)}$  times so the faster  
19 rate holds for continuous-time GD. However, discrete-time GD  $R(t) < (1 - \alpha^{2(L-1)}\eta/2)^t R(0)$  requires  $\eta \lesssim 1/\alpha^{2(L-1)}$ ,  
20 thus the number of iterations may not exponentially decrease. (2) (general case of rectangular weight matrices) This  
21 can be achieved by padding zeros as long as  $\min\{d_1, \dots, d_{L-2}\} \geq \min\{d_0, d_{L-1}\}$ . Actually, Our analysis works for  
22 a general initialization. Let  $m = \min\{d_{L-1}, d_0\}$  and  $A = U\Sigma V$  where  $U \in \mathbb{R}^{d_{L-1} \times m}$ ,  $V \in \mathbb{R}^{d_0 \times m}$ ,  $\Sigma \in \mathbb{R}^{m \times m}$  be  
23 the singular value decomposition of  $A$ . We can initialize

$$W_L = 0; W_{L-1} \simeq U\Sigma^{1/(L-1)}, W_{L-2} \simeq \Sigma^{1/(L-1)}, \dots, W_2 \simeq \Sigma^{1/(L-1)}, W_1 \simeq \Sigma^{1/(L-1)}V, \quad (2)$$

24 where the symbol “ $\simeq$ ” stands for equality up to zero-valued paddings. This initialization is similar as the Procedure 1 in  
25 (Arora et al. ICLR2019), but with the top layer to be zero.

26 **For Reviewer 3:** (1) (all layers are  $d \times d$ ) Our proof only relies on the *dynamic invariance* and top layer to be  
27 zero. So the result also holds for the general case,  $\min\{d_1, \dots, d_{L-1}\} \geq \min\{d_0, d_L\}$ , in which the matrices are  
28 not necessarily square. We will clarify this in the revised version. (2) (direct consequence of a known alignment  
29 property...) This known property actually has been widely used in the previous works (Bartlett et al. ICML2018,  
30 Arora et al. ICLR2019, Shamir COLT2019, Du et al. ICML 2019) for analyzing the optimization of linear networks,  
31 but coming up with the right initialization to fully utilize this property is not straightforward. That is why the global  
32 convergence of GD for general linear networks has not been established until this submission. Especially, the picture of  
33 symmetry break behind the ZAS initialization could be useful for analyzing linear networks in other setting, such as  
34 matrix factorization, binary classification, etc.. (3) (empirical results on how this specific initialization may help in  
35 practice) We have conducted some experiments, and please refer to the beginning of this rebuttal for the results. (4)  
36 (results on deep linear nets (e.g. Ji and Telgarsky, 2019)) Thanks for pointing out this reference and we will add it to the  
37 related work section. This work studies the properties of solutions that the GD converges to, without providing any  
38 convergence rate. The ZAS initialization might help to establish the convergence in their setting.

39 **For Reviewer 4:** (1) (...the development of a new initialization method is useful unless its shown to be competitive  
40 on real problems) We have done some experiments for real problems (given in the beginning of this reresponse).  
41 and the results suggest that the ZAS-type initialization is use-  
42 ful for nonlinear ResNets in practice. However, we want to  
43 stress that the goal of this submission is to provide theoret-  
44 ical understanding of the optimization of deep linear nets.  
45 The ZAS initialization is proposed to obtain a global conver-  
46 gence guarantee of GD for optimizing deep linear nets. (2)  
47 (more convincing in any case to show these curves for multiple  
48 runs...) Please see right figure, which shows results of multiple  
49 runs. We will add it in the revised version. (3) (The paper is  
50 rather poorly written) We apologize for the confusion caused  
51 by the writing. We will improve it in the revised version.

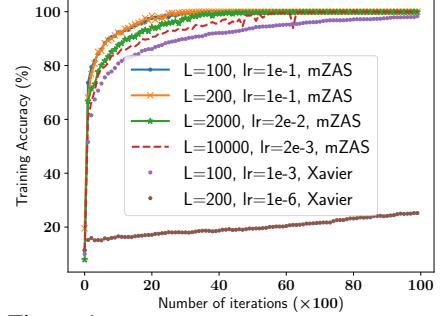


Figure 1: The comparison of training curves between two initializations. The learning rate is manually tuned to achieve the best convergence performance. The curves of GD with Xavier initialization for  $L=2000, 10000$  are not shown, since they always blow up.

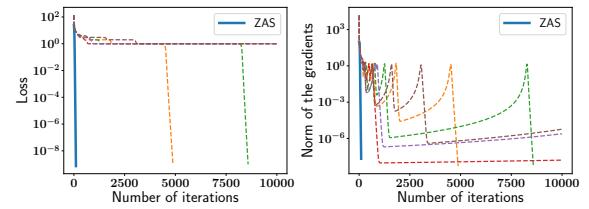


Figure 2: The five dashed lines correspond to the multiple runs of GD with the Xavier initialization. It is shown that GD successfully escape the saddle region for only 2 out of 5 times in the given number of iterations.