

Supplement for the paper “Tractable Bayesian
Network Structure Learning with Bounded
Vertex Cover Number”

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Data	n	Unbounded		tree-width = 1		tree-width = 2		tree-width = 3		tree-width = 4		tree-width = 5	
		Score	Time	Score	Time	Score	Time	Score	Time	Score	Time	Score	Time
Mildew10000	35	-407644	1	-424311	34	-407821(0.32%)	>14400	-407644	1446	-407644	789	-407644	1723
Mildew1000	35	-47102	0	-47245	21	-47102	34	-47102	384	-47102	645	-47102	1104
Mildew100	35	-5720	2	-6000	1	-5935	2	-5786(1.15%)	>14400	-5746(0.46%)	>14400	-5720	2985
Water10000	32	-128706	26	-129804	26	-129103(0.26%)	>14400	-129016(0.3%)	>14400	-128803(0.13%)	>14400	-128709(0.06%)	>14400
Water1000	32	-13262	31	-13307	43	-13276(0.13%)	>14400	-13264(0.04%)	>14400	-13262	3102	-13262	4365
Water100	32	-1501	28	-1508	17	-1502(0.03%)	>14400	-1501(0.06%)	>14400	-1501	5578	-1501	7870
alarm10000	37	-105227	786	-118383	67	-110217(5.53%)	>14400	-106349(2.59%)	>14400	-106797(3.07%)	>14400	-106975(3.34%)	>14400
alarm1000	37	-11240	7	-12315	69	-11774(4.93%)	>14400	-11469(2.48%)	>14400	-11291(0.95%)	>14400	-11273(0.94%)	>14400
alarm100	37	-1349	2	-1473	70	-1443(5.57%)	>14400	-1455(7.84%)	>14400	-1412(4.79%)	>14400	-1401(4.24%)	>14400
asia10000	8	-22466	1	-23037	1	-22469(0.14%)	>14400	-22466	541	-22466	510	-22466	31
asia1000	8	-2317	0	-2372	0	-2329(0.55%)	>14400	-2317	159	-2317	111	-2317	6
asia100	8	-246	0	-246	0	-246	2	-246	21	-246	42	-246	2
carpo10000	60	-174131	6739	-181686	839	-177127(0.89%)	>14400	-175324(8.63%)	>14400	-175312(10.84%)	>14400	-175324(11.37%)	>14400
carpo1000	60	-17719	106	-18413	1142	-18192(2.29%)	>14400	-18436(10.82%)	>14400	-18271(10.42%)	>14400	-19112(17.27%)	>14400
carpo100	60	-1829	900	-1974	465	-2050(10.71%)	>14400	-2065(13.95%)	>14400	-2101(21.17%)	>14400	-2131(23.02%)	>14400
haifinder10000	56	-497632	86	-512293	557	-500054(0.7%)	>14400	-499777(9.44%)	>14400	-500313(11.75%)	>14400	-500313(16.56%)	>14400
haifinder1000	56	-52473	16	-53401	617	-52478(0.19%)	>14400	-52593(7.37%)	>14400	-52616(10.98%)	>14400	-52605(12.36%)	>14400
haifinder100	56	-6019	0	-6076	41	-6021	182	-6019	7926	-6034(5.87%)	>14400	-6041(7.25%)	>14400
insurance10000	27	-132969	23	-144844	24	-142329(6.77%)	>14400	-138442(4.35%)	>14400	-133866(1.07%)	>14400	-133108(0.51%)	>14400
insurance1000	27	-13887	1	-14702	15	-14261(2.63%)	>14400	-14170(2.04%)	>14400	-13887	785	-13887	1314
insurance100	27	-1686	0	-1728	10	-1711(1.42%)	>14400	-1692(0.35%)	>14400	-1686	341	-1686	492
kreditfamily	18	-16696	0	-16698	0	-16696	1	-16696	32	-16696	33	-16696	26
Abalone	9	-15401	1	-16278	0	-15462	36	-15417(0.1%)	>14400	-15401	420	-15401	457
adult15N	15	-351151	2	-356324	2	-354806(0.81%)	>14400	-354656(0.94%)	>14400	-351712(0.16%)	>14400	-351151	283
Flag	29	-2748	7	-2803	13	-2748	374	-2796(1.7%)	>14400	-2780(1.15%)	>14400	-2756(0.29%)	>14400
Heart	23	-2397	1	-2445	8	-2397	86	-2401(0.16%)	>14400	-2397	239	-2397	413
Hepatitis	20	-1323	1	-1338	12	-1323	45	-1330(0.6%)	>14400	-1323	232	-1323	184
Horse	28	-4525	3	-4579	11	-4561(0.81%)	>14400	-4525	781	-4525	844	-4525	1153
Housing	14	-3080	33	-3479	2	-3424(5.01%)	>14400	-3251(3.29%)	>14400	-3247(5.34%)	>14400	-3150(3.44%)	>14400
Voting	17	-4643	1	-4658	5	-4643	30	-4643	174	-4643	193	-4643	163
Wine	14	-1271	2	-1285	2	-1271	34	-1271	134	-1271	249	-1271	127
Zoo	17	-848	0	-858	2	-848	15	-848	70	-848	118	-848	101

Table 1: Results for unbounded tree-width and tree-widths 1–5. For each case, the score of an optimal DAG and the running time is reported; if the computations were not finished at the time limit, we report the score of the best DAG found and the gap (the gap is a ratio $|s_1 - s_2|/|s_1|$, where s_1 is the score of the best feasible solution and s_2 is the smallest known upper bound for the score of an optimal network). Scores and running times are rounded to the nearest integer.

Data	n	Unbounded		VC number = 1		VC number = 2		VC number = 3		VC number = 4		VC number = 5	
		Score	Time	Score	Time	Score	Time	Score	Time	Score	Time	Score	Time
Mildew10000	35	-407644	1	-509428	0	-473918	3	-454469	3	-446114	13	-439382	11
Mildew1000	35	-47102	0	-54412	0	-51411	1	-49772	1	-49202	10	-48759	4
Mildew100	35	-5720	2	-6309	0	-6151	0	-5983	74	-5892	57	-5833	70
Water10000	32	-128706	26	-160387	0	-154095	1	-148486	7	-142990	45	-138879	53
Water1000	32	-13262	31	-16339	0	-15754	1	-15186	1	-14643	1	-14215	1
Water100	32	-1501	28	-1791	0	-1732	0	-1682	2	-1633	1	-1588	1
alarm10000	37	-105227	786	-172927	1	-154994	5	-137775	38	-130289	86	-124761	625
alarm1000	37	-11240	7	-17884	1	-16072	2	-14402	6	-13744	22	-13264	86
alarm100	37	-1349	2	-2037	0	-1883	2	-1747	2	-1648	6	-1578	10
asia10000	8	-22466	1	-25794	0	-22820	2	-22469	10	-22466	21	-22466	20
asia1000	8	-2317	0	-2621	0	-2357	0	-2320	3	-2317	6	-2317	8
asia100	8	-246	0	-271	0	-247	0	-246	1	-246	1	-246	1
carpo10000	60	-174131	6739	-220979	2	-201424	16	-191893	141	-186470	481	-182037	843
carpo1000	60	-17719	106	-22109	1	-20244	8	-19397	19	-18913	24	-18546	34
carpo100	60	-1829	900	-2420	4	-2213	7	-2104	34	-2055	151	-2005	310
hailfinder10000	56	-497632	86	-604656	4	-586718	14	-571259	41	-558578	73	-547059	141
hailfinder1000	56	-52473	16	-62208	1	-60701	5	-59224	7	-57975	9	-56897	14
hailfinder100	56	-6019	0	-6788	0	-6664	1	-6543	2	-6437	2	-6339	1
insurance10000	27	-132969	23	-190711	0	-172051	3	-162410	27	-154919	104	-147054	29
insurance1000	27	-13887	1	-19193	0	-17671	1	-16830	12	-15997	7	-15239	9
insurance100	27	-1686	0	-2102	0	-1988	5	-1913	3	-1843	10	-1793	5
kreditfamily	18	-16696	0	-17213	0	-16978	0	-16821	0	-16762	0	-16725	1
Abalone	9	-15401	1	-16408	0	-15775	13	-15571	34	-15474	49	-15401	44
adult15N	15	-351151	2	-396233	0	-369029	4	-357371	15	-355010	79	-353333	154
Flag	29	-2748	7	-3007	0	-2933	1	-2880	6	-2844	72	-2820	288
Heart	23	-2397	1	-2867	0	-2746	1	-2631	4	-2531	4	-2476	8
Hepatitis	20	-1323	1	-1414	0	-1374	9	-1352	16	-1339	18	-1331	21
Horse	28	-4525	3	-4872	0	-4763	0	-4674	6	-4628	22	-4589	7
Housing	14	-3080	33	-3715	0	-3494	10	-3400	433	-3292	680	-3198	819
Voting	17	-4643	1	-4885	0	-4703	13	-4670	16	-4658	21	-4653	30
Wine	14	-1271	2	-1313	0	-1289	23	-1276	35	-1272	27	-1271	25
Zoo	17	-848	0	-971	0	-927	9	-890	6	-863	5	-850	4

Table 2: Results for unbounded vertex cover number and vertex cover numbers 1–5. For each case, the score of an optimal DAG and the running time is reported; if the computations were not finished at the time limit, we report the score of the best DAG found and the gap (the gap is a ratio $|s_1 - s_2|/|s_1|$, where s_1 is the score of the best feasible solution and s_2 is the smallest known upper bound for the score of an optimal network). Scores and running times are rounded to the nearest integer.

		VC number = 6		VC number = 7		VC number = 8		VC number = 9		VC number = 10	
Data	n	Score	Time	Score	Time	Score	Time	Score	Time	Score	Time
Mildew10000	35	-434370	15	-429812	15	-425404	20	-421147	25	-416998	15
Mildew1000	35	-48349	4	-47945	4	-47580	3	-47449	13	-47321	13
Mildew100	35	-5804	192	-5779	367	-5755	162	-5742	220	-5735	146
Water10000	32	-135480	64	-132971	44	-131218	42	-130068	90	-129113	76
Water1000	32	-13896	4	-13689	8	-13508	9	-13391	17	-13303	50
Water100	32	-1548	2	-1535	18	-1523	14	-1516	15	-1509	12
alarm10000	37	-119562	417	-115699	872	-113166	2012	-111372	13304	-109791	13620
alarm1000	37	-12757	37	-12449	180	-12168	106	-11983	123	-11792	159
alarm100	37	-1535	33	-1510	104	-1476	37	-1449	42	-1422	83
asia10000	8	-22466	34	-22466	44	-22466	41	-22466	42	-22466	47
asia1000	8	-2317	5	-2317	6	-2317	6	-2317	6	-2317	6
asia100	8	-246	1	-246	1	-246	1	-246	1	-246	1
carpo10000	60	-178440	1624	-175924	1925	-174860	2271	-174658	10174	-174476	9054
carpo1000	60	-18167	28	-17933	82	-17804	94	-17780	348	-17756	428
carpo100	60	-1969	785	-1933	796	-1913	3612	-1907(0.6%)	>14400	-1899(0.74%)	>14400
hailfinder10000	56	-539233	632	-531427	226	-524603	232	-519114	340	-514429	290
hailfinder1000	56	-56117	23	-55355	20	-54770	42	-54191	33	-53749	47
hailfinder100	56	-6278	5	-6218	4	-6160	4	-6125	4	-6096	6
insurance10000	27	-142526	118	-139026	152	-136489	181	-135456	244	-134443	493
insurance1000	27	-14745	16	-14398	19	-14183	21	-14074	35	-14001	55
insurance100	27	-1763	8	-1742	12	-1720	9	-1708	11	-1701	19
kreditfamily	18	-16699	1	-16696	0	-16696	0	-16696	0	-16696	0
Abalone	9	-15401	57	-15401	45	-15401	39	-15401	40	-15401	40
adult15N	15	-351891	52	-351470	51	-351243	52	-351151	48	-351151	47
Flag	29	-2802	270	-2783	296	-2773	365	-2767	344	-2761	247
Heart	23	-2452	17	-2428	14	-2418	26	-2409	27	-2403	24
Hepatitis	20	-1327	44	-1323	17	-1323	24	-1323	27	-1323	22
Horse	28	-4568	14	-4557	50	-4546	31	-4539	52	-4530	35
Housing	14	-3135	788	-3111	3130	-3088	5622	-3080	2690	-3080	1393
Voting	17	-4647	36	-4643	30	-4643	21	-4643	33	-4643	22
Wine	14	-1271	23	-1271	26	-1271	21	-1271	29	-1271	19
Zoo	17	-849	9	-848	8	-848	9	-848	12	-848	6

Table 3: Results for vertex cover numbers 6–10. For each case, the score of an optimal DAG and the running time is reported; if the computations were not finished at the time limit, we report the score of the best DAG found and the gap (the gap is a ratio $|s_1 - s_2|/|s_1|$, where s_1 is the score of the best feasible solution and s_2 is the smallest known upper bound for the score of an optimal network). Scores and running times are rounded to the nearest integer.