MAP estimation in Binary MRFs via Bipartite Multi-cuts (Supplementary Material)

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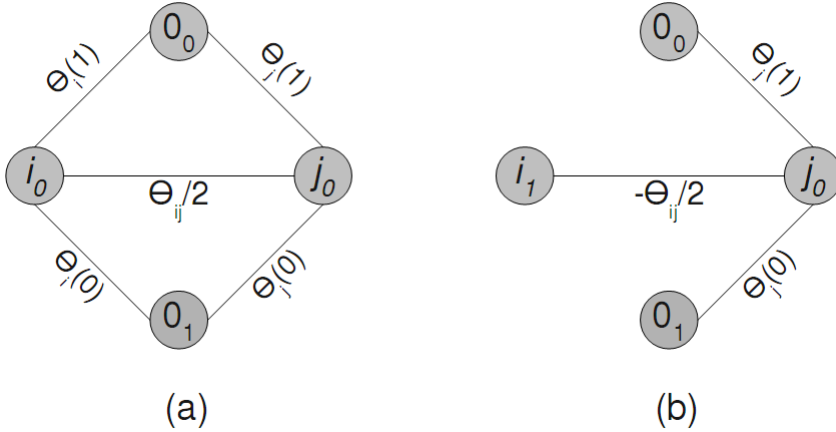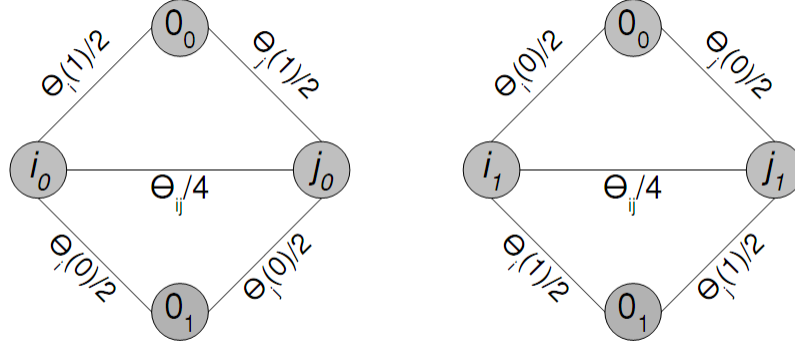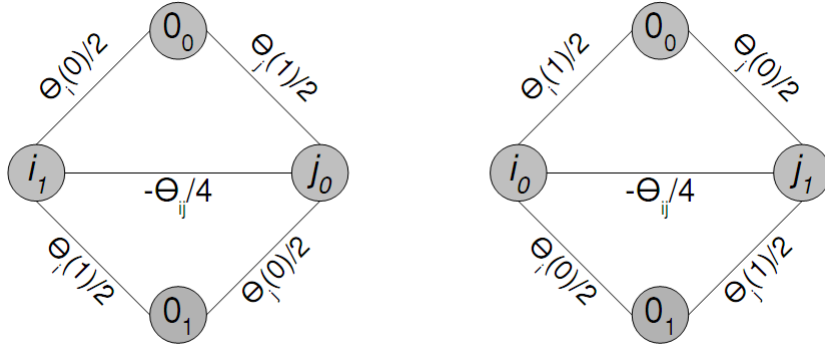sundar@cse.iitb.ac.in

1   Optimized Graph Construction

Figure 1: *(a) The graph construction for submodular edges. (b) The graph construction for non-submodular edges. Here $\theta_{ij} = \theta_{ij}(0,1) + \theta_{ij}(1,0) - \theta_{ij}(0,0) - \theta_{ij}(1,1)$. If the edge is submodular, θ_{ij} is non-negative. We have also assumed $\Theta \geq 0$. Hence all the edges in this part of the graph have non-negative edge weights.*

(a) Submodular edges

(b) Non-Submodular edges

Figure 2: *(a) Symmetric graph construction. Here $\theta_{ij} = \theta_{ij}(0,1) + \theta_{ij}(1,0) - \theta_{ij}(0,0) - \theta_{ij}(1,1)$. We have also assumed $\Theta \geq 0$.*

2 Symmetric Graph Construction

3 Relationship with Cycle Inequalities

Theorem 1. *Suppose we construct our symmetric minimum cut graph H with all possible edges $(i_s, j_t) \forall s, t \in \{0,1\}$ for every edge $(i,j) \in \mathcal{E}$, instead of two that we currently get due to zero-normalized edge potentials. Then, BMC-Sym LP along with the constraints $d_{i_s j_t} + d_{i_s j_{\bar{t}}} = 1$ $\forall (i_s, j_t) \in \mathcal{E}_H$ is equivalent to cycle inequality LP.*

Proof. Consider a cycle $C \in \mathcal{C}$ in S. Suppose $F \subseteq C$ such that $|F|$ is odd. We will show that the constraints $\sum_{e \in F}(1 - d_e) + \sum_{e \in C \setminus F} d_e \geq 1$ in the cycle inequality LP is equivalent to constraints $\sum_e d'_e \geq 1$ in BMC-Sym LP for set of paths $\mathcal{P}_{(C,F)}$, which we will now define. Consider the following procedure to obtain $\mathcal{P}_{(C,F)} \subseteq \mathcal{P}$. Recall $\mathcal{P}$ is the set of all paths between ST pairs in H. Suppose we start with node i_0^1 in C and move along the cycle C i.e C is $i_0^1 - i_0^2 - \ldots - i_0^{|C|} - i_0^1$. Now, consider the path P $i_{s_1}^1 - i_{s_2}^2 - \ldots i_{s_{|C|}}^{|C|} - i_{s_{|C|+1}}^1$ in H. Here, $s_l \in \{0, 1\}$ and $s_l \neq s_{l+1}$ iff $(i_0^l, i_0^{l+1}) \in F$. It is not hard to see that $s_1 \neq s_{|C|+1}$ since $|F|$ is odd. Therefore, the considered path $P \in \mathcal{P}$. Applying the same procedure with different starting nodes leads to different paths in H. We denote the set of all such paths as $\mathcal{P}_{(C,F)}$. Here, $\sum_{e \in F}(1 - d_e) + \sum_{e \in C \setminus F} d_e \geq 1$ is equivalent to $\sum_{e \in P} d'_e \geq 1$, where $P \in \mathcal{P}_{(C,F)}$, since $d'_e = d'_{\overline{e}}$ and $d'_{i_s j_t} + d'_{i_s j_{\overline{t}}} = 1 \, \forall (i_s, j_t) \in \mathcal{E}_H$. This equivalence shows that every cycle constraint corresponds to a set of equivalent path constraints in BMC-Sym LP and vice-versa. It is easy to see that the two LPs have the same objective value. Hence, two LPs are equivalent. $\qquad\square$