A Additional results

In this section, we propose an empirical study of uncertainty quality over standard regression benchmarks such as the UCI datasets presented in [8, 18, 19]. Similarly to Figure 3, in Table 2 we have reported the mean absolute error between the amount of data points above or equal to a certain predicted probability and the ideal one. As shown here, UMAL still produces proper and calibrated conditional distribution.

Table 2: Mean and standard deviation for all folds of the mean absolute error between the predicted calibration and the perfect ideal calibration.

	Housing	Concrete	Energy	Kin8nm	Naval	Power	Protein	Wine	Yacht
N	$\mathbf{.08 \pm .04}$	$\mathbf{.03 \pm .01}$	$\mathbf{.03 \pm .01}$	$\mathbf{.02 \pm .01}$	$.39 \pm .02$	$\mathbf{.02 \pm .01}$	$.03 \pm .00$	$\mathbf{.03 \pm .01}$	$\mathbf{.06 \pm .02}$
L	$\mathbf{.07 \pm .04}$	$\mathbf{.05 \pm .02}$	$\mathbf{.04 \pm .01}$	$\mathbf{.04 \pm .01}$	$.35 \pm .03$	$.05 \pm .01$	$\mathbf{.02 \pm .00}$	$\mathbf{.06 \pm .02}$	$\mathbf{.07 \pm .02}$
I-QR	$.20 \pm .05$	$.18 \pm .02$	$.15 \pm .04$	$.17 \pm .01$	$\mathbf{.12 \pm .05}$	$.20 \pm .02$	$.06 \pm .01$	$.19 \pm .03$	$.14 \pm .06$
2N-MDN	$\mathbf{.07 \pm .05}$	$\mathbf{.04 \pm .02}$	$\mathbf{.05 \pm .02}$	$\mathbf{.01 \pm .01}$	$.36 \pm .04$	$\mathbf{.03 \pm .01}$	$.06 \pm .01$	$.08 \pm .03$	$.06 \pm .02$
3N-MDN	$\mathbf{.07 \pm .05}$	$\mathbf{.07 \pm .03}$	$.04 \pm .02$	$\mathbf{.02 \pm .01}$	$.37 \pm .04$	$\mathbf{.03 \pm .01}$	$.11 \pm .01$	$.15 \pm .04$	$\mathbf{.07 \pm .02}$
4N-MDN	$\mathbf{.09 \pm .05}$	$.10 \pm .03$	$\mathbf{.05 \pm .02}$	$\mathbf{.03 \pm .01}$	$.36 \pm .05$	$\mathbf{.03 \pm .01}$	$.15 \pm .01$	$.18 \pm .03$	$\mathbf{.07 \pm .04}$
10N-MDN	$.12 \pm .06$	$.22 \pm .06$	$.09 \pm .04$	$.08 \pm .01$	$.33 \pm .05$	$\mathbf{.03 \pm .01}$	$.22 \pm .01$	$.18 \pm .02$	$\mathbf{.09 \pm .05}$
2L-MDN	$\mathbf{.09 \pm .05}$	$\mathbf{.06 \pm .02}$	$\mathbf{.05 \pm .02}$	$\mathbf{.04 \pm .01}$	$.32 \pm .04$	$.07 \pm .02$	$.07 \pm .00$	$\mathbf{.06 \pm .02}$	$\mathbf{.06 \pm .03}$
3L-MDN	$\mathbf{.11 \pm .05}$	$.10 \pm .03$	$.08 \pm .03$	$.05 \pm .01$	$.29 \pm .04$	$.08 \pm .02$	$.12 \pm .01$	$.16 \pm .03$	$\mathbf{.06 \pm .02}$
4L-MDN	$\mathbf{.14 \pm .06}$	$.12 \pm .03$	$\mathbf{.08 \pm .04}$	$.06 \pm .01$	$.31 \pm .04$	$.07 \pm .02$	$.15 \pm .01$	$.15 \pm .02$	$\mathbf{.05 \pm .02}$
10L-MDN	$.21 \pm .05$	$.18 \pm .04$	$.16 \pm .05$	$.11 \pm .01$	$.27 \pm .06$	$.08 \pm .02$	$.22 \pm .01$	$.17 \pm .01$	$\mathbf{.10 \pm .04}$
I-$\mathcal{A}$LD	$\mathbf{.07 \pm .06}$	$\mathbf{.04 \pm .01}$	$\mathbf{.05 \pm .02}$	$\mathbf{.04 \pm .01}$	$.44 \pm .01$	$\mathbf{.04 \pm .01}$	$.07 \pm .00$	$\mathbf{.03 \pm .01}$	$\mathbf{.09 \pm .04}$
UMAL	$\mathbf{.10 \pm .05}$	$\mathbf{.07 \pm .04}$	$\mathbf{.06 \pm .02}$	$\mathbf{.02 \pm .01}$	$.43 \pm .01$	$\mathbf{.02 \pm .01}$	$\mathbf{.02 \pm .00}$	$.13 \pm .06$	$\mathbf{.06 \pm .03}$

For the sake of completeness, we have also computed the UMAL negative log-likelihood for UCI datasets (see Table 3) following [19]. These results re-emphasise that UMAL is in the best positions. However, it should be noted that most of these databases have a **small sample size** and that aleatoric uncertainty cannot be reliably estimated in this regime. We hypothesize that a better solution would be to simultaneously estimate epistemic (as in [18, 19, 8]) and aleatoric uncertainty.

Table 3: Comparison of the Negative Mean Log-Likelihood of the test set over different train-test folds proposed in [8].

	Housing	Concrete	Energy	Kin8nm	Naval	Power	Protein	Wine	Yacht
N	$2.76 \pm .34$	$\mathbf{3.20 \pm .16}$	$\mathbf{2.13 \pm .24}$	$\mathbf{-1.15 \pm .03}$	$-3.67 \pm .01$	$\mathbf{2.83 \pm .03}$	$2.84 \pm .03$	$1.05 \pm .14$	$\mathbf{1.86 \pm .31}$
L	$\mathbf{2.59 \pm .20}$	$\mathbf{3.21 \pm .13}$	$\mathbf{2.06 \pm .20}$	$\mathbf{-1.08 \pm .04}$	$\mathbf{-3.73 \pm .04}$	$\mathbf{2.87 \pm .03}$	$2.74 \pm .01$	$1.00 \pm .08$	$\mathbf{1.54 \pm .37}$
I-QR	10.96 ± 2.4	$10.19 \pm .95$	9.45 ± 1.3	$9.22 \pm .66$	$5.14 \pm .89$	$8.39 \pm .45$	$8.14 \pm .52$	$12.30 \pm .91$	10.32 ± 2.9
2N-MDN	$2.74 \pm .30$	$3.25 \pm .21$	$\mathbf{2.02 \pm .30}$	$\mathbf{-1.15 \pm .05}$	$-3.66 \pm .02$	$\mathbf{2.85 \pm .05}$	$2.56 \pm .03$	$1.33 \pm .61$	$\mathbf{1.55 \pm .32}$
3N-MDN	$2.68 \pm .28$	$3.64 \pm .28$	$\mathbf{2.30 \pm .43}$	$-1.15 \pm .05$	$-3.66 \pm .01$	$\mathbf{2.85 \pm .04}$	$2.90 \pm .15$	0.69 ± 1.0	$\mathbf{1.54 \pm .52}$
4N-MDN	$2.87 \pm .46$	$3.74 \pm .28$	$2.46 \pm .39$	$\mathbf{-1.12 \pm .04}$	$\mathbf{-3.66 \pm .03}$	$\mathbf{2.86 \pm .05}$	$3.32 \pm .11$	$\mathbf{0.52 \pm .90}$	$\mathbf{1.43 \pm .36}$
10N-MDN	$3.10 \pm .46$	5.64 ± 1.1	$3.03 \pm .71$	$-0.99 \pm .06$	$-3.64 \pm .03$	$\mathbf{2.86 \pm .04}$	$4.94 \pm .75$	$0.75 \pm .95$	$\mathbf{1.75 \pm .49}$
2L-MDN	$2.61 \pm .23$	$\mathbf{3.28 \pm .14}$	$\mathbf{2.06 \pm .30}$	$\mathbf{-1.10 \pm .04}$	$-3.70 \pm .06$	$2.91 \pm .05$	$2.50 \pm .03$	$0.59 \pm .63$	$\mathbf{1.37 \pm .42}$
3L-MDN	$2.65 \pm .25$	$3.45 \pm .16$	$\mathbf{2.30 \pm .21}$	$\mathbf{-1.09 \pm .03}$	$\mathbf{-3.66 \pm .06}$	$2.95 \pm .04$	$2.65 \pm .06$	$\mathbf{-0.81 \pm .70}$	$\mathbf{1.39 \pm .35}$
4L-MDN	$2.76 \pm .42$	$3.57 \pm .14$	$\mathbf{2.31 \pm .35}$	$\mathbf{-1.10 \pm .05}$	$\mathbf{-3.68 \pm .06}$	$2.93 \pm .04$	$2.79 \pm .08$	$\mathbf{-0.65 \pm .96}$	$\mathbf{1.45 \pm .35}$
10L-MDN	$3.17 \pm .46$	$3.95 \pm .34$	$2.80 \pm .49$	$-0.98 \pm .07$	$-3.62 \pm .10$	$2.96 \pm .05$	$3.46 \pm .12$	$0.52 \pm .74$	$\mathbf{1.63 \pm .34}$
I-$\mathcal{ALD}$	$2.79 \pm .56$	$3.87 \pm .12$	$\mathbf{2.28 \pm .11}$	$-1.00 \pm .05$	$-2.82 \pm .01$	$\mathbf{2.89 \pm .02}$	$2.68 \pm .01$	$1.01 \pm .07$	$\mathbf{1.78 \pm .41}$
UMAL	$\mathbf{2.59 \pm .26}$	$3.74 \pm .15$	$\mathbf{2.13 \pm .14}$	$\mathbf{-1.09 \pm .03}$	$-2.81 \pm .01$	$\mathbf{2.85 \pm .03}$	$\mathbf{2.40 \pm .01}$	$\mathbf{0.14 \pm .70}$	$\mathbf{1.41 \pm .38}$