

1 We would like to thank the three reviewers for their time and valuable suggestions. We are also glad that all reviewers  
2 have found the results of this paper significant.

---

4 **Response to Reviewer #2:** We agree that polytrees can still be considered a limited class of networks, but this class  
5 is more general compared to rooted tree models since polytrees can model fusion of information from independent  
6 sources as opposed to rooted trees. There exist a large number of articles in the literature that support the importance of  
7 studying tree structured networks since they have been shown to be useful in practice, often as an approximation of  
8 more complex networks as in [9, 10] and also [M. B. Eisen, et al., PNAS, vol. 95, no. 25, pp. 14863–14868, 1998].

9 We are very glad that Reviewer #2 has highlighted in his/her comments that our method requires only statistics of the  
10 second and third order and found it as an interesting property.

11 Reviewer #2 correctly points out that the collapsed quasi-skeleton is not formally defined. The collapsed quasi-skeleton  
12 is the collapsed representation of the quasi-skeleton of a graph. We didn't provide a formal definition because we  
13 assumed the expression to be self-explanatory. However, we gladly accept the reviewer's suggestion of adding a formal  
14 definition for this concept to improve clarity.

15 We also agree with the reviewer that the wording of step 3 of the algorithm can definitely be improved. We would like  
16 to clarify here, though, that by saying “the hidden clusters partially overlap” we mean that the hidden clusters in each  
17 rooted subtree have at least one hidden node in common (as mentioned in Section 3.3). There are examples of these  
18 situations in the article. For example, the polytree of Figure 3 demonstrates an occurrence of overlap between hidden  
19 clusters: the hidden node  $y_{h_6}$  is shared between the hidden cluster of the rooted subtree with root  $y_{h_3}$  and the hidden  
20 cluster of the rooted subtree with root  $y_1$ . In case of acceptance of the paper, we will make sure to include additional  
21 examples, to make the article easier to read.

22 We will also definitely work on improving the clarity of the overall article, of course including a rewording of the  
23 sentence which mentions “ancestral graphs” in the abstract, and adding the orientations in Figure 1(b).

---

25 **Response to Reviewer #3:** We think that the overall relevance of the article comes from the combination of the  
26 positive result (“if the degree assumptions on the hidden nodes are verified, then we can successfully recover the  
27 model”) with the negative result (“if the degree assumptions are not verified, then there is a polytree model with the  
28 same observable nodes and fewer hidden nodes”). When the degree assumptions are not met, our method selects a  
29 polytree with a minimal number of hidden nodes. In other words, following a form of Occam's razor principle, our  
30 algorithm always introduces the least number of unobserved nodes to explain the observations. Thus, the theoretical  
31 relevance of the results is that either the network is recovered correctly or the simplest network (in the sense of number  
32 of hidden nodes) that explains the data is selected. We find a property of this kind quite relevant in application scenarios,  
33 too, given that Occam's razor is arguably one of the cardinal principles in all sciences. Indeed, another consequence of  
34 the negative result is that, with no additional information/observations, no test can be constructed to determine if the  
35 degree conditions are met or not.

36 However, there are also practical situations where the degree conditions are automatically guaranteed: for example in  
37 phylogenetic trees when all the leaves (i.e., current species) are observed [P. Erdos, et al., “A few logs suffice to build  
38 (almost) all trees”, 1999].

39 In case of acceptance of the article, these points will be incorporated in the abstract and introduction in order to provide  
40 more motivation and context for our approach. We will also follow Reviewer #3's other recommendation to introduce  
41 an empirical validation of our method.

---

43 **Response to Reviewer #5:** We thank Reviewer #5 for the positive comments on the importance of this contribution.  
44 However, we would like to clarify that the proposed algorithm does not require to know the number of hidden nodes  
45 in advance. The only information required for the learning process is the observations of the observable nodes. The  
46 exact number and location of the hidden nodes (no matter how many hidden nodes exist in the network) is inferred  
47 by the algorithm under the assumption that certain degree conditions are satisfied on each hidden node. When these  
48 degree conditions are not verified for at least one hidden node, we show that there exists another polytree with the same  
49 observable nodes and fewer hidden nodes. Thus, our algorithm either recovers the polytree correctly or selects the  
50 simplest polytree (in the sense of number of hidden nodes) that explains the observed data.

51 We are more than happy to follow Reviewer #5's suggestion (shared also by Reviewer #3) to include an empirical  
52 validation of our technique.

---