We appreciate the time invested in carefully reading our paper and the very helpful and detailed comments by all reviewers. We will address their concerns in the revised version and thank the reviewers for pointing out that: our work is very clearly written, structured (R1) and detailed (R2), with a clear and convenient notation and a sound math (R1), and provides new links between and new interpretations of well-known graph-based procedures (R1, R2, R3).

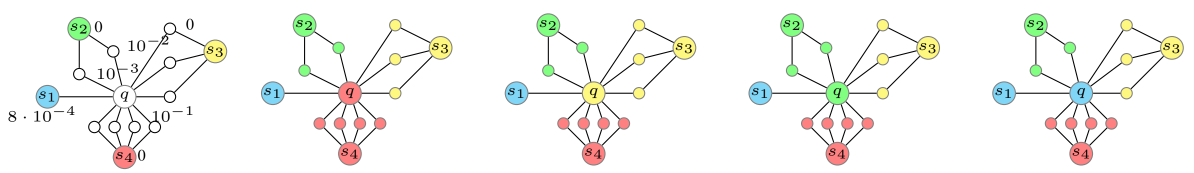
The numbered citations will refer to the references of the paper.

Reproducibility (R2): The Random Walker and Watershed algorithms are well-known and implemented, e.g., in scikit-image's `segmentation.random_walker` and vigra's `analysis.watersheds`. Based on R2's comment, we will make the code and the edge-weights used for our illustrative computations available on GitHub for easy reproducibility.

Connection to machine learning (R2): In lines 11ff of the paper we state that seeded segmentation is essentially the same as graph-based semi-supervised learning, thus presenting our work as an instance of transductive learning. Given a limited number of labelled observations, the categories of the unlabelled data are inferred from their relation to the labelled data. We also cite machine learning literature that uses the Random Walker algorithm (and thus our Probabilistic Watershed) under different names in lines 49f [5, 40, 41] or combines it with deep leaning [9, 37]. We are happy to elaborate these connections further in the revised version of the paper.

Empirical support (R3): Indeed the focus of our paper is conceptual and theoretical, as pointed out in the introduction (line 45). We have rigorously proven the equivalence of the Probabilistic Watershed to the Random Walker (theorem 4.1) and that a limit case calculates the potentials of the Power Watershed (theorem 5.1). The empirical effectiveness of the Random Walker and the Power Watershed algorithm *and thus of the equivalent Probabilistic Watershed* has been thoroughly established when these algorithms were introduced [13, 23, 41] and when they were subsequently applied. For instance, the Random Walker has been recently applied in the references below and in [9, 37]. In view of this vast empirical support, we decided not to add further experiments but to concentrate on deepening the theoretical insight in (connections between) different graph-based seeded segmentation algorithms, thus placing our contribution in the "Theory" subject area of NeurIPS 2019.

Potential practical implications (R1 + R3): Although practical implications were not the focus of our work, the improved understanding of the aforementioned graph-based seeded segmentation algorithms and the introduced techniques may lead to new practical algorithms. An example of a new point of view provided by our work is the entropy regularised spanning forest sampling. It explains the interpolation between a Voronoi tessellation, the Random Walker and the Watershed algorithm discussed in [13] by varying the temperature of the Gibbs-distribution over all spanning forests. We have included an illustration of the temperature's effect in Figure 1, which we will also add to the appendix of the paper. Another idea inspired by our work would be to apply the Probabilistic Watershed framework to directed graphs, where one would sample directed spanning forests with the seeds as sinks to segment the unlabelled nodes. This might lead to a new practical algorithm for semi-supervised learning on directed graphs such as social/citation or Web networks and could be related to directed random walks.



(a) Graph with four seeds (b) $\mu = 0$ (c) $\mu = 10$ (d) $\mu = 100$ (e) $\mu \to \infty$, Watershed

Figure 1: Effect of the inverse temperature μ on Probabilistic Watershed solutions. 1a shows a graph with 4 seeds and edge costs, $c(e)$. All paths from the query node q to a seed s_i have the same cost, (only indicated once per seed). 1b - 1d show the Probabilistic Watershed's segmentation for edge weights $\exp(-\mu\, c(e))$. As μ grows, q's assignment changes from a weight-independent one over two Random Walker assignments to the Watershed assignment.

References

Bockelmann, N. et al. (2019). Sparse annotations with random walks for U-Net segmentation of biodegradable bone implants in synchrotron microtomograms. In *International Conference on Medical Imaging with Deep Learning – Extended Abstract Track*.

Bui, V. et al. (2018). An automatic random walk based method for 3d segmentation of the heart in cardiac computed tomography images. In *ISBI*.

Liu, Z. et al. (2018). A method for PET-CT lung cancer segmentation based on improved random walk. In *ICPR*.