Supplementary material of "Balancing Efficiency and Fairness in On-Demand Ridesourcing"

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Proof of Lemma 3.2

Proof. Based on the reassignment procedure described in REASSIGN, the vehicles $\mathcal{V}$ can be divided into several subsets $\mathcal{S} = \{S_1, S_2, \dots, S_t\}$, where S_i consists of all vehicles that participate in the chain swapping (line 6-10 of REASSIGN) in one iteration. We assume $\mathcal{S}$ is nonempty, since otherwise we have $M_{\text{new}} = M_{\text{old}}$. Note that (1) if a vehicle v appears in S_i in some iteration, it will be assigned to $M_{\text{fair}}(v)$ after that iteration and will never appear again in S_j for any $j > i$. Hence we have $S_i \cap S_j = \emptyset$ for any $i \neq j$; (2) there might be vehicles who do not participate in any swapping procedure. Hence $\bigcup_i S_i$ may not necessarily equal to $\mathcal{V}$. We define the set of vehicles $\mathcal{V}_o = \mathcal{V} \setminus \bigcup_{1 \le i \le t} S_i$. Note that $\forall v \in \mathcal{V}_o$, $w_{v,M_{\text{new}}(v)} = w_{v,M_{\text{old}}(v)}$.

We further define $p_i = |S_i|$ and $\mathcal{E}_i(M)$ as the *partial efficiency* of vehicles in S_i of the assignment M , i.e. $\mathcal{E}_i(M) = \sum_{v \in S_i} (h_v + w_{v,M(v)})$.

We focus on an arbitrary set S_i. When $p_i = 1$, it trivially holds that $\mathcal{E}_i(M_{\text{new}}) \ge \mathcal{E}_i(M_{\text{old}})$. When $p_i \ge 2$, in the following we quantify how much efficiency loss occurs during the swapping.

Let us first define the set of p_i vehicles $\{v_j\}_{1 \le j \le p_i}$ indexed based on the swapping order, such that $M_{\text{new}}(v_j) = M_{\text{old}}(v_{j+1}), 1 \le j < p_i$. Thus, we have

$$\begin{aligned} \mathcal{E}_i(M_{\text{old}}) - \mathcal{E}_i(M_{\text{new}}) &= w_{v_1, M_{\text{old}}(v_1)} - w_{v_{p_i}, M_{\text{new}}(v_{p_i})} + \sum_{2 \le j \le p_i} \left(w_{v_j, M_{\text{old}}(v_j)} - w_{v_{j-1}, M_{\text{old}}(v_j)} \right) \\ &\le w_{v_0, M_{\text{old}}(v_0)} - w_{v_{p_i-1}, M_{\text{new}}(v_{p_i-1})} + (p_i - 1)\Delta \\ &\le f + (p_i - 1)\Delta \end{aligned} \tag{1}$$

Next, we know from REASSIGN that every vehicle v in the swapping chain is reassigned to request $M_{\text{fair}}(v)$ in the output assignment M_{new}. Thus, we have

$$\mathcal{E}_i(M_{\text{new}}) = \sum_{v \in S_i} \left(h_v + w_{v, M_{\text{new}}(v)} \right) \ge \sum_{v \in S_i} \mathcal{F}_{\text{opt}} \ge p_i \mathcal{F}_{\text{opt}}$$

This implies

$$\mathcal{E}(M_{\text{new}}) \ge \sum_{1 \le i \le |\mathcal{S}|} \mathcal{E}_i(M_{\text{new}}) \ge 2|\mathcal{S}| \cdot \mathcal{F}_{\text{opt}} \Rightarrow |\mathcal{S}| \le \frac{\mathcal{E}(M_{\text{new}})}{2\mathcal{F}_{\text{opt}}} \tag{2}$$

Let us now consider all available vehicles $v \in \mathcal{V}$. For simplicity, we define the set of vehicles $\mathcal{V}_o = \mathcal{V} \setminus \bigcup_{1 \le i \le t} S_i$. Therefore, from equation (1) and (2), and the fact that for every $v \in \mathcal{V}_o$,

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$w_{v,M_{\text{new}}(v)} = w_{v,M_{\text{old}}(v)}$, we have

$$\begin{aligned}
\mathcal{E}(M_{\text{old}}) - \mathcal{E}(M_{\text{new}}) &= \sum_{1\le i\le|\mathcal{S}|} (\mathcal{E}_i(M_{\text{old}}) - \mathcal{E}_i(M_{\text{new}})) \\
&\le \sum_{1\le i\le|\mathcal{S}|} (f + (p_i - 1)\Delta) \\
&\le |\mathcal{S}| \cdot f + n\Delta \\
&\le \frac{f \cdot \mathcal{E}(M_{\text{new}})}{2\mathcal{F}_{\text{opt}}} + n\Delta
\end{aligned}$$

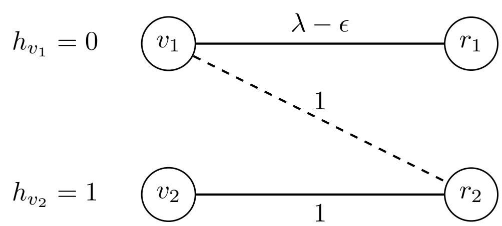
Rearrange the terms in the last inequality and we obtain

$$\mathcal{E}(M_{\text{new}}) \ge \frac{2\mathcal{F}_{\text{opt}}}{2\mathcal{F}_{\text{opt}} + f}(\mathcal{E}(M_{\text{old}}) - n\Delta)$$

which is exactly what stated in the lemma. □

Proof of Theorem 3.3

Proof. For any $0 \le \lambda \le 1$ and $\alpha > \frac{2}{2+\lambda}$, consider the following problem instances.



Here ϵ is set as $\min\{\frac{1}{2}(2 + \lambda - \frac{2}{\alpha}), \lambda\}$. Because $\alpha > \frac{2}{2+\lambda}$, this guarantees $\lambda \ge \epsilon > 0$.

Note that this problem instance has $\Delta = 0$. There are only two feasible assignments in this instance:

- the *efficient assignment* M_{eff} (marked by solid lines) assigns r_1 to v_1 and r_2 to v_2 and gives $\mathcal{E}_{\text{opt}} = 2 + \lambda - \epsilon$;
- the *fair assignment* M_{fair} (marked by dashed lines) assigns r_2 to v_1 and leaves r_1 unmatched, and gives $\mathcal{F}_{\text{opt}} = 1$.

Note that among these two assignments, M_{fair} is the only one with fairness value at least λ, and we have

$$\frac{\mathcal{E}(M_{\text{fair}})}{\mathcal{E}_{\text{opt}} - n\Delta} = \frac{2}{2 + \lambda - \epsilon} < \frac{2}{2 + \lambda - (2 + \lambda - \frac{2}{\alpha})} = \alpha.$$

Thus in this problem instance, any assignment that satisfies the fairness requirement stated in the lemma cannot satisfy the efficiency requirement. □