

1 Thanks to all the reviewers for their thorough feedback and valuable suggestions.

## 2 **Reviewer 1**

3 We will add more intuitions and pictures to make the proofs in our appendix clearer.

## 4 **Reviewer 2**

5 *Typos:* We will fix the typos in Definitions 1 (i.e.  $h_j/2$  for layer  $j$ ) and 3 (i.e. largest number instead of smallest).

6 *Justifying our stability assumptions:* For neural networks trained with standard algorithms, noise stability was previously  
7 observed in [Marcos et al. 2018, Arora et al. 2018]. While there is no formal proof showing any optimization algorithm  
8 must find a noise stable solution, there is evidence that the solutions found are indeed noise stable. Our own experiments  
9 also show that the paths constructed using these properties can indeed connect two different solutions.

10 *Purpose of  $\theta_2$  through  $\theta_{d-1}$ :* Our path construction for Theorem 1 passes through each  $\theta_i$ , not just  $\theta_1$ . For example, in  
11 Figure 1 Step (2) uses  $\theta_2$  and Step (4) uses  $\theta_1$ . We describe how to connect each  $\theta_i$  to  $\theta_{i-1}$  in our proof of Lemma 1.  
12 We will add some helpful figures in the revised version that should hopefully clarify to our path construction.

13 *Properties in Definition 2:* It is better to have larger  $\mu$ . This definition is exactly the same as in [Arora et al. 2018], due  
14 to space limitations we chose to focus on properties that are slightly different.

15 *Definition 4, question about activation contraction  $c$ :* Main confusion here is that the noise stability quantities are  
16 defined *specifically* for ReLU networks. For ReLU activations, the constant  $c$  is mostly related to the fraction of neurons  
17 that are active. The definitions would need to be different for other activations. While there are a lot of terms in  
18 Definition 4, the overall message is that larger layer cushion and larger interlayer cushion leads to better noise stability  
19 (i.e. smaller  $\epsilon$ ). We also note that the theory is asymptotic, and that the numbers computed in a real network might be  
20 loose; however our experimental results suggest that the theory does indeed lead to paths connecting different solutions.

21 *Explaining interlayer smoothness, “cirular” argument:* At a high level, interlayer smoothness assumes the network is  
22 close to its *linear* approximation, but what we need to prove is that the network output is nearly *constant*. A large class  
23 of functions have good linear approximations, but not all linear functions are constants. Only in combination with our  
24 other noise stability assumptions can we show that the dropped out network has similar loss as the original network.

25 *Experiments on CNNs:* As explained in Remark 1, our dropout-based path constructions naturally extend to convolutional  
26 nets since we can think of each channel as one hidden unit. All our noise stability properties also apply to convolutional  
27 nets (as in [Arora et al. 2018]). We chose to show experiments with convolutional nets because these architectures are  
28 widespread and of practical interest.

29 *Figure 2:* Figure 2 is meant to construct a path based on Theorem 3 (instead of Theorem 1 which requires dropping-out  
30 1/2 the units). In Theorem 3 we show that if there exists a small network with low loss, then one only needs to drop out  
31 a smaller fraction of the units in each hidden layer. The left plot shows the error under different levels of dropout; the  
32 middle plot shows that connecting a network with its dropout version with  $p = 0.2$  has almost constant loss; the right  
33 plot shows that there exists a neural network whose size is 0.2 times the original network that has low loss. Combining  
34 these three plots with Theorem 3, we know that there exists a path with almost constant loss. We also note that data in  
35 Figure 2 are from networks NOT trained with dropout. We will clarify these points in the final version.

36 *Comparison to interpolation:* Interpolating results in substantially higher loss than our path construction (loss/accuracy  
37 1.61/67.2% on MNIST and 2.34/10% on CIFAR). We will add a plot in the revision. Thank you for the suggestion!

## 38 **Reviewer 3**

39 *Typos:* We will fix the typos in Definition 1 (same one pointed out by Reviewer 2), Lemma 2 and the construction of our  
40 counterexample. Thanks for pointing these out!

41 *What we get from interlayer smoothness:* Interlayer smoothness does not in and of itself ensure that the output of the  
42 network is stable as we discuss in our response to Reviewer 2. Requiring the property to hold for all  $t$  is crucial for us  
43 to be able to directly interpolate between the original network and the fully dropped out version using a single linear  
44 segment. Note however that even if we only assume the property holds at the endpoint of this path we would still be  
45 able to connect the original network to its dropped out version, but doing so would come at the cost of needing more  
46 segments to construct the path as we did in Lemma 1.

47 *Figure 3:* The leftmost plot in Figure 3 is simply meant to give readers a sense for what the values for each of the  
48 different components that comprise our ultimate definition of noise stability tend to look like in practice: the quantities  
49 appearing in the denominator of noise stability are reasonable constants bounded away from zero, and conversely those  
50 appearing in the numerator are not too large. It is true that the bounds are asymptotic and directly computing the bound  
51 may not give a good number on real data, but our experiments show that the paths constructed are reasonable (although  
52 they are not the same as the paths constructed in [Garipov et al. 2018, Draxler et al. 2018]).