1 Appendix: Notational equivalence to the Yu and colleagues flanker model

Yu et al. use the following notation for their update:

$$P(s_2, M \mid X_t) = \frac{p(s_t \mid s_2, M)p(s_2, M \mid \mathbf{X}_{t-1})}{\sum_{s_2', M} p(s_t' \mid s_2, M)p(s_2', M \mid \mathbf{X}_{t-1})} \tag{1}$$

In their notation, the stimulus array is indexed such that s_2 is the target and $s_{1,3}$ are the flankers. Therefore, their s_2 is simply our G. Their M is a trial compatibility or congruence variable, taking on the values of I(ncongruent) and C(ongruent). This gives a straightforward remapping from their joint probability space over target identity and congruence into our space of context and target:

Stimulus	C	G == s_2	M
SSS	S_S	S	Congruent
HHH	H_H	H	Congruent
SHS	S_S	H	Incongruent
HSH	H_H	S	Incongruent

Their prior $P(s_2, M \mid \mathbf{X}_{t-1})$ is equivalent to our prior (it is simply the posterior from the previous timestep). Their input x_t is an input vector concatenating the input vectors from the target and two flankers, $[x_1, x_2, x_3]$, such that:

$$x_1(t) \sim (N)(\alpha_1\mu_1 + \alpha_2\mu_2, \sigma_1^2 + \sigma_2^2) \tag{2}$$
$$x_2(t) \sim (N)(\alpha_1\mu_2 + \alpha_2\mu_1 + \alpha_2\mu_3, \sigma_1^2 + 2\sigma_2^2) \tag{3}$$
$$x_3(t) \sim (N)(\alpha_1\mu_3 + \alpha_2\mu_2, \sigma_1^2 + \sigma_2^2) \tag{4}$$

Since the two flanker stimuli are always identical in this experiment, we can define $\mu_c := \mu_1 = \mu_3$ and $\mu_g := \mu_2$. Next, we divide the means by α_1, and map $\frac{\alpha_2}{\alpha_1} := \alpha_m u$ to make $x_2(t)$ equivalent to e^G. Since the three likelihoods are multiplied and the two flanker likelihoods are identical, updating jointly on $[x_1, x_3]$ will be equivalent to updating twice on two draws of e^C.

Yu and colleagues also summarize their prior by defining β to be the prior probability of a congruent trial. We can define the priors in the following way to reflect this:

$$P_0(C = c_0, G = g_0) = \frac{\beta}{2} \tag{5}$$
$$P_0(C = c_0, G = g_1) = \frac{1-\beta}{2} \tag{6}$$
$$P_0(C = c_1, G = g_0) = \frac{1-\beta}{2} \tag{7}$$
$$P_0(C = c_1, G = g_1) = \frac{\beta}{2} \tag{8}$$

2 Full derivation of AX-CPT log likelihood expressions

In the internal context AX-CPT, $t_{c^{on}} \neq t_{g^{on}}$, so we index context samples using ℓ and target samples using t. We therefore define $l_t(t_x) = P(e_t^G \mid G = g_x)$ and $l_\ell(c_x) = P(e_\ell^C \mid C = c_x)$ for the likelihoods, with $x \in \{0, 1\}$ indexing stimuli. We can write the log likelihood for the two responses to the symmetric AX-CPT, divide numerator and denominator by the product of g_1 and c_1 likelihoods, and then rewrite the log likelihood ratios into the z term that evolve as biased Wiener processes in the continuum limit. Note that here the context and target walks start at different times.

$$\log Z = \log \frac{P_0(C=c_0,G=g_0)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_0)\prod_{t=t_g^{on}}^{\tau} l_t(g_0) + P_0(C=c_1,G=g_1)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_1)\prod_{t=t_g^{on}}^{\tau} l_t(g_1)}{P_0(C=c_0,G=g_1)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_0)\prod_{t=t_g^{on}}^{\tau} l_t(g_1) + P_0(C=c_1,G=g_0)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_1)\prod_{t=t_g^{on}}^{\tau} l_t(g_0)} \tag{9}$$

$$= \log \frac{P_0(C=c_0,G=g_0)\prod_{\ell=t_c^{on}}^{\tau} \frac{l_\ell(c_0)}{l_\ell(c_1)}\prod_{t=t_g^{on}}^{\tau} \frac{l_t(g_0)}{l_t(g_1)} + P_0(C=c_1,G=g_1)}{P_0(C=c_0,G=g_1)\prod_{\ell=t_c^{on}}^{\tau} \frac{l_\ell(c_0)}{l_\ell(c_1)} + P_0(C=c_1,G=g_0)\prod_{t=t_g^{on}}^{\tau} \frac{l_t(g_0)}{l_t(g_1)}} \tag{10}$$

$$= \log \frac{P_0(C=c_0,G=g_0)\sum_{\ell=t_c^{on}}^{\tau} \log\frac{l_\ell(c_0)}{l_\ell(c_1)}\sum_{t=t_g^{on}}^{\tau} \log\frac{l_t(g_0)}{l_t(g_1)} + P_0(C=c_1,G=g_1)}{P_0(C=c_0,G=g_1)\sum_{\ell=t_c^{on}}^{\tau} \log\frac{l_\ell(c_0)}{l_\ell(c_1)} + P_0(C=c_1,G=g_0)\sum_{t=t_g^{on}}^{\tau} \log\frac{l_t(g_0)}{l_t(g_1)}} \tag{11}$$

$$= \log \frac{P_0(C=c_0,G=g_0)e^{z_c^\tau}e^{z_g^\tau} + P_0(C=c_1,G=g_1)}{P_0(C=c_0,G=g_1)e^{z_c^\tau} + P_0(C=c_1,G=g_0)e^{z_g^\tau}} \tag{12}$$

We can do the same for the asymmetric variant.

$$\log Z = \log \frac{P_0(C=c_0,G=g_0)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_0)\prod_{t=t_g^{on}}^{\tau} l_t(g_0)}{\begin{aligned}P_0(C=c_0,G=g_1)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_0)\prod_{t=t_g^{on}}^{\tau} l_t(g_1) + P_0(C=c_1,G=g_0)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_1)\prod_{t=t_g^{on}}^{\tau} l_t(g_0)\\ + P_0(C=c_1,G=g_1)\prod_{\ell=t_c^{on}}^{\tau} l_\ell(c_1)\prod_{t=t_g^{on}}^{\tau} l_t(g_1)\end{aligned}} \tag{13}$$

$$= \log \frac{P_0(C=c_0,G=g_0)\prod_{\ell=t_c^{on}}^{\tau} \frac{l_\ell(c_0)}{l_\ell(c_1)}\prod_{t=t_g^{on}}^{\tau} \frac{l_t(g_0)}{l_t(g_1)}}{P_0(C=c_0,G=g_1)\prod_{\ell=t_c^{on}}^{\tau} \frac{l_\ell(c_0)}{l_\ell(c_1)} + P_0(C=c_1,G=g_0)\prod_{t=t_g^{on}}^{\tau} \frac{l_t(g_0)}{l_t(g_1)} + P_0(C=c_1,G=g_1)} \tag{14}$$

$$= \log \frac{P_0(C=c_0,G=g_0)\sum_{\ell=t_c^{on}}^{\tau} \log\frac{l_\ell(c_0)}{l_\ell(c_1)}\sum_{t=t_g^{on}}^{\tau} \log\frac{l_t(g_0)}{l_t(g_1)}}{P_0(C=c_0,G=g_1)\sum_{\ell=t_c^{on}}^{\tau} \log\frac{l_\ell(c_0)}{l_\ell(c_1)} + P_0(C=c_1,G=g_0)\sum_{t=t_g^{on}}^{\tau} \log\frac{l_t(g_0)}{l_t(g_1)} + P_0(C=c_1,G=g_1)} \tag{15}$$

$$= \log \frac{P_0(C=c_0,G=g_0)e^{z_c^\tau}e^{z_g^\tau}}{P_0(C=c_0,G=g_1)e^{z_c^\tau} + P_0(C=c_1,G=g_0)e^{z_g^\tau} + P_0(C=c_1,G=g_1)} \tag{16}$$