Deterministic Single-Pass Algorithm for LDA –Supporting Material–

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Theorem 1. *If ϵ and ν exist satisfying $0 < \epsilon < S_j < \nu$ for any j,*

$$\eta_j = \frac{S_j}{\tau + \sum_d^j S_d} \tag{1}$$

satisfies

$$\lim_{j \to \infty} \eta_j = 0, \ \sum_j^{\infty} \eta_j = \infty, \ \sum_j^{\infty} \eta_j^2 < \infty \tag{2}$$

Proof. If ϵ and ν exist satisfying $0 < \epsilon < S_j < \nu$,

$$\frac{\epsilon}{\tau + j\nu} < \eta_j = \frac{S_j}{\tau + \sum_d^j S_d} < \frac{\nu}{\tau + j\epsilon}. \tag{3}$$

Moreover, it is known (H.Robbins and S.Monro, 1951) that the following stepsize satisfies the conditions in Eq. (2),

$$\eta_j = \frac{\tau_1}{\tau_2 + j} \ (\tau_1, \tau_2 > 0). \tag{4}$$

Therefore, since

$$\lim_{j \to \infty} \frac{\epsilon}{\tau + j\nu} = 0, \ \lim_{j \to \infty} \frac{\nu}{\tau + j\epsilon} = 0, \tag{5}$$

$$\sum_j^{\infty} \frac{\epsilon/\nu}{\tau/\nu + j} = \infty, \ \sum_j^{\infty} \frac{\nu/\epsilon}{\tau/\epsilon + j} = \infty, \tag{6}$$

the squeeze theorem shows

$$\lim_{j \to \infty} \frac{S_j}{\tau + \sum_d^j S_d} = 0, \ \sum_j^{\infty} \frac{S_j}{\tau + \sum_d^j S_d} = \infty. \tag{7}$$

Also

$$\sum_j^{\infty} \left(\frac{S_j}{\tau + \sum_d^j S_d} \right)^2 < \sum_j^{\infty} \left(\frac{\nu/\epsilon}{\tau/\epsilon + j} \right)^2 < \infty. \tag{8}$$

$\square$

References

H.Robbins and S.Monro. A stochastic approximation method. In *Annals of Mathematical Statistics*, pages 400–407, 1951.